REFLECTIVE CHARACTERISTICS OF NATURAL WATERS: THE ACCURACY OF SELECTED APPROXIMATIONS

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INTRODUCTION

One of the most popular models intensively used in ocean optics is Gordon's quasi-single-scattering approximation (QSSA)[1-3]. This approximation describes relationships between measured apparent optical properties such as the reflectance just above air-sea interface and inherent optical properties (e. g., the absorption coefficient a and the backscattering coefficient b_b). The idea of QSSA is based on the fact that for a scattering phase function highly peaked in the forward direction, the most part of the scattered light remains in the beam, and the loss is due to absorption and backscattering. Also it is assumed that the probability of photon absorption is not small. Therefore, as it was shown in some of recent publications[4, 5], this approximation is limited presumably by clear oceanic and moderately-turbid waters and can be used for highly-turbid waters only as a rough estimate. It was shown by comparison with the more accurate Hulst's approximation[4, 6] and with the exact radiative transfer calculations[5, 7] that in turbid waters with a strong scattering, errors of calculations using QSSA can reach 50% and even more.

In the present paper we study the accuracy of QSSA and also other approximations for three different reflection characteristics of natural waters as compared to exact radiative transfer calculations.

BASIC DEFINITIONS

The reflection function (RF) R is defined as the ratio of the intensity of light (radiance) reflected from a given medium to that for an absolutely white Lambertian surface under the same conditions of illumination and viewing. From this definition follows that RF just below the air-water interface equivalent to $\pi L_u/(\mu_1 E_c)$, where L_u is the reflected upwelling radiance, E_c is the incident collimated irradiance, and μ_1 is the cosine of the incidence angle θ_1 (just below the surface). The plane albedo (PA) $\Re$ is defined as the integral of the reflection function $R(\mu_1, \mu_2, \varphi)$:

$$\Re(\mu_1) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu_1, \mu_2, \varphi)\mu_2 d\mu_2 d\varphi, \qquad (1)$$

where μ_2 is the cosine of the observation angle θ_2 and φ is the relative azimuth. We see that the value of $\Re$ gives the diffuse reflectance under directional illumination of a turbid layer. The spherical albedo, SA, (another term is irradiance ratio) r is determined as:

$$r = 2\int_0^1 \Re(\mu_1)\mu_1 d\mu_1 . \qquad (2)$$

Thus, the physical meaning of r is the same as $\Re$ except for the diffuse light illumination conditions. Below we give results of calculations on all 3 mentioned characteristics, assuming that the scattering layer is a plane-parallel, optically homogeneous and infinite.

REFLECTION FUNCTION

The exact calculation of RF is rather difficult, because of necessity to take into account a large number of parameters, including the scattering geometry. Due to this reason, the development of a suitable approximation is also a very difficult task. On this reason, we consider below only QSSA; for any observation it is expressed by equation[1, 7]:

$$R(\omega_0, F, \mu_1, \mu_2) = \frac{\omega_0 p(\theta)}{4(1-\omega_0 F)(\mu_1 + \mu_2)}, \tag{3}$$

where $p(\theta)$ is a given scattering phase function and F is a forward scattering probability (i.e., F = 1-B, where B is a backscattering probability). In our simulations we represent $p(\theta)$ as a finite sum:

$$p(\theta) = \sum_{i=0}^{N} x_i P_i(\theta), \tag{4}$$

where $P_i(\theta)$ are the Legendre polynomials of order i and x_i are the expansion coefficients for a given phase function. Two different phase functions simulating extreme natural waters conditions[4, 9]: (1) very clear waters with g = 0.5033 and B = 0.1559 and (2) very turbid waters with g = 0.9583 and B = 0.008692 have been selected for this study (Table 1 and Fig. 1). They have been calculated using Mie theory for the gamma particle size distribution $f(a) = A\exp(-9a/a_{ef})$ for different values of the effective radius a_{ef} and refractive indices $m = n - i\chi$ as specified in Table 1. Here A is the normalization constant $\left(\int_0^\infty f(a)\,da = 1\right)$ and a is the radius of a spherical particle. Values of the backscattering probability, calculated for the well-known Henyey-Greenstein (HG) phase function with the above-mentioned values of g also presented in the Table 1; the HG phase functions with these parameters of g and B are shown in Fig. 1 for comparison.

Table 1. Parameters used in Mie calculations and results for the pair (g, B). Values of B calculated for HG phase functions are shown in the last column in the brackets.

a_{ef}, μm	n	χ	g	B
0.116	1.25	0.001	0.5033	0.1559 (0.1692)
5	1.2	0.01	0.9583	0.008692 (0.009005)

Below we show (Figs. 2, 3) results only for main phase functions, selected values of the single-scattering albedo ω_0, and normal observation (μ_2 = 1). Note, that exact modeling was performed using the radiative transfer code as it was described by Mishchenko et al.[10] For the convenience, we have made calculations using the cosine (μ_0) of the solar zenith angle θ_0 instead of μ_1 [transformation was performed by Snell's law: $\mu_0 = \sqrt{1 - n_w^2(1-\mu_1^2)}$, where refractive index of water (n_w) was taken as 1.333]. The error is below 26.5% for the case studied in Figs. 2a and 3a (g = 0.5033) at $\omega_0 \le 0.6$ for all incidence angles; the error is below 27.7% for the case studied in Figs. 2b and 3b (g = 0.9583) at $\omega_0 \le 0.9$ for almost all incidence angles (except the cases of $\theta_1 \rightarrow 0$, where glint and backscattering enhancement effects may play a role and also at $\theta_1 \rightarrow \pi/2$). The increase of the error for larger values of ω_0 is due to the fact that the QSSA is better suitable for weakly scattering media.

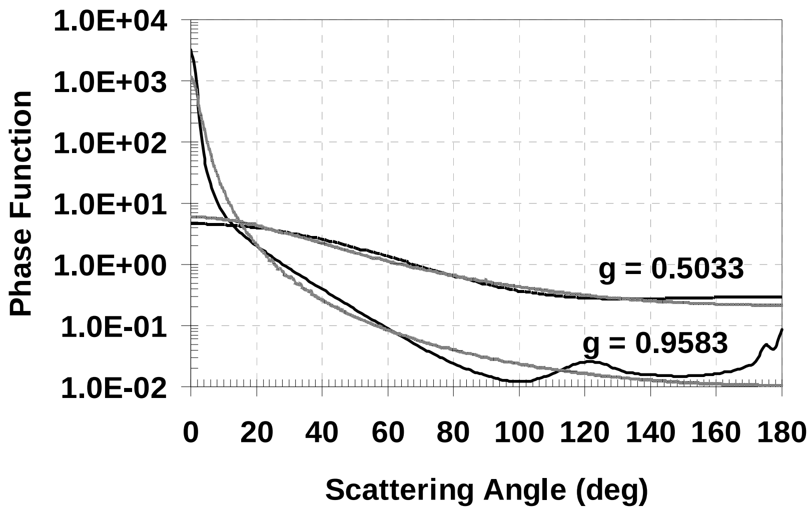

Fig. 1. Phase functions (black curves) used for calculations; Henyey-Greenstein phase functions (with the same values of *g*, pale curves) are given for comparison.

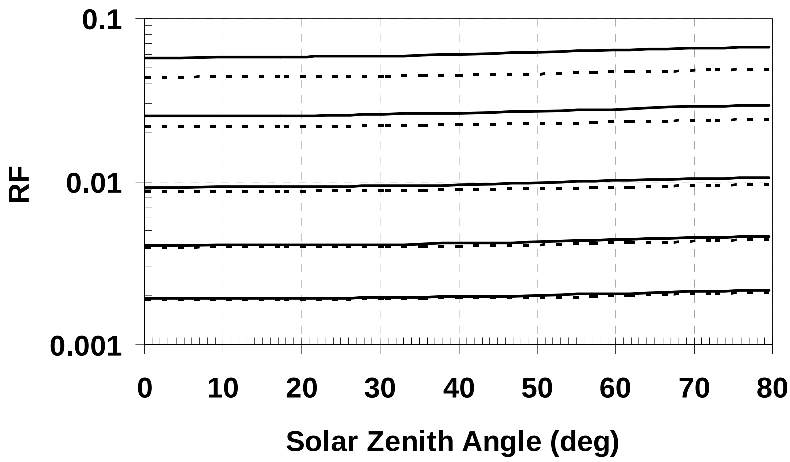

Fig. 2.a. Reflection function (RF) obtained using exact radiative transfer calculations (solid curves) and QSSA, Eq. (3) (dotted curves) for phase function with g = 0.5033 and B = 0.1559 at normal observation and ω_0 = 0.05, 0.1, 0.2, 0.4, 0.6 (from the bottom to up).

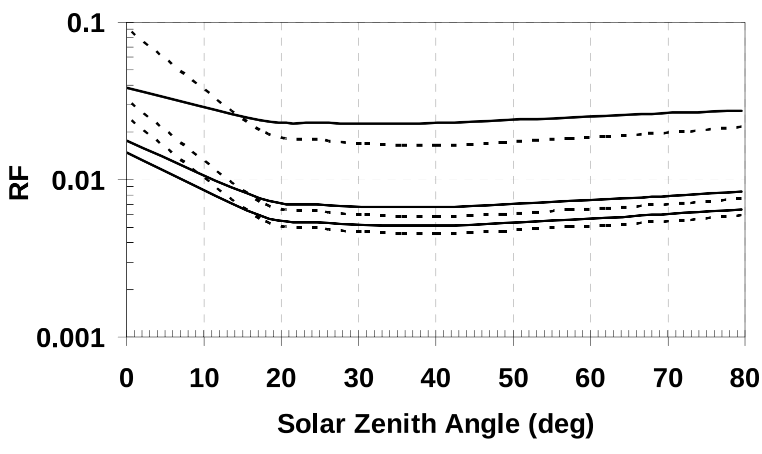

Fig. 2b. The same as Fig. 2a, but for phase function with g = 0.9583 and B = 0.008692 at ω_0 = 0.7, 0.75, 0.9 (from the bottom to up).

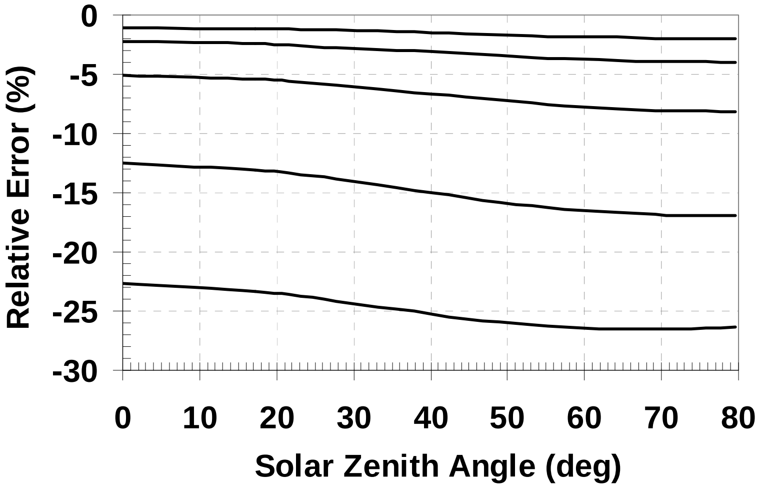

Fig. 3a. Relative errors of Eq. (3) as compared to exact radiative transfer calculations with *g* = 0.5033 and *B* = 0.1559 at ω_0 = 0.05, 0.1, 0.2, 0.4, 0.6 (from up to the bottom).

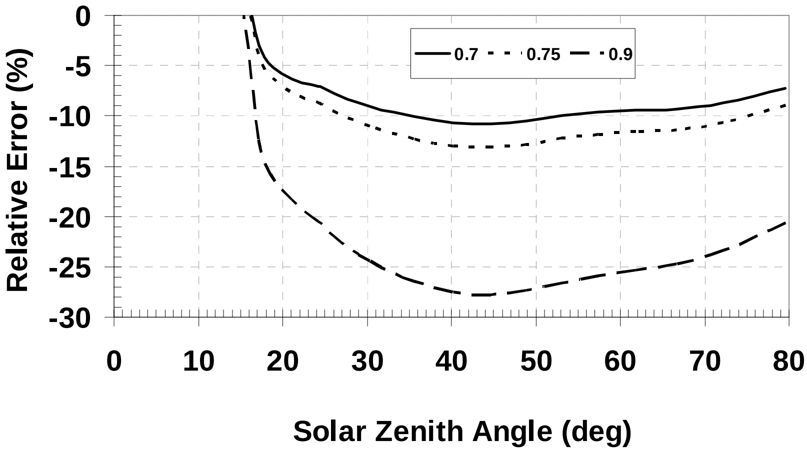

Fig. 3b. The same as Fig. 3a, but for *g* = 0.9583 and *B* = 0.008692 at ω_0 = 0.7, 0.75, 0.9.

PLANE ALBEDO

For the plane albedo, QSSA approximation can be reduced to the following form[7]:

$$\Re(\omega_0, F, \mu_1) = \frac{\omega_0}{2(1-\omega_0 F)} \sum_{i=0}^{N} (-1)^i x_i P_i(\mu_1) Q_i(\mu_1), \tag{5}$$

where

$$Q_i(\mu_1) = \int_0^1 \frac{P_i(\mu_2)\mu_2 d\mu_2}{\mu_1 + \mu_2}. \tag{6}$$

For the high values of g ($0.85 \le g \le 0.96$) and $\omega_0 > 0.7$, the following approximation has been found[11]:

$$\Re(g, \omega_0, \mu_1) = F_{cor}(\mu_1, g) \frac{1-s}{1+2\mu_1 s}, \tag{7}$$

where s is the Hulst's similarity parameter[6], defined as

$$s = \sqrt{\frac{1-\omega_0}{1-g\omega_0}}, \tag{8}$$

$F_{cor}(\mu_1, g)$ is the corrected factor expressed by the equation

$$F_{cor}(\mu_1, g) = \exp\left[\left(A_1\zeta + A_2\zeta^2\right)s + \left(A_3\zeta + A_4\zeta^2\right)s^2\right],\ \zeta = \mu_1 - 0.5, \tag{9}$$

and $A_i = \sum_{j=0}^{2} \alpha_{ij} g^j$, where elements α_{ij} are determinate from the following matrix:

$$\hat{\alpha} = \begin{pmatrix} 9.994 & -24.697 & 15.455 \\ 8.203 & -14.708 & 5.302 \\ 13.359 & -24.799 & 7.543 \\ 2.514 & -5.156 & 6.100 \end{pmatrix} \tag{10}$$

using fitting procedure. Results are presented in Figs. 4 and 5.

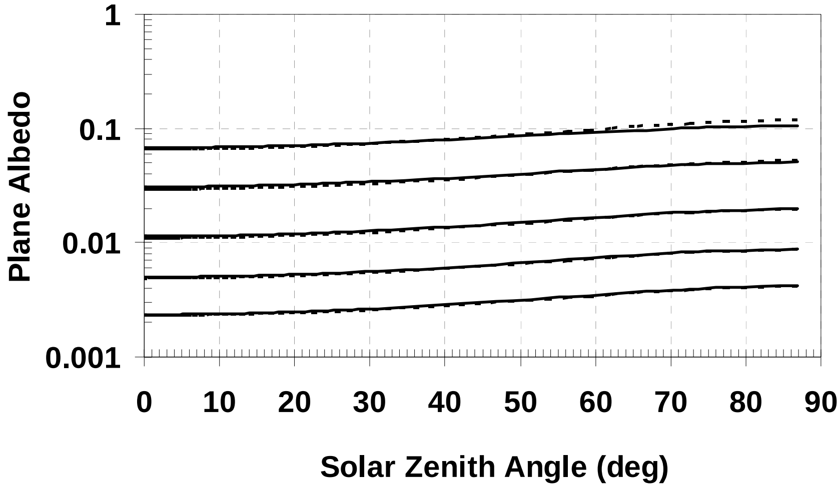

Fig. 4a. Plane albedo obtained using exact radiative transfer calculations (solid curves) and QSSA, Eq. (5) (dotted curves) with g = 0.5033 and B = 0.1559 at ω_0 = 0.05, 0.1, 0.2, 0.4, 0.6 (from the bottom upwards).

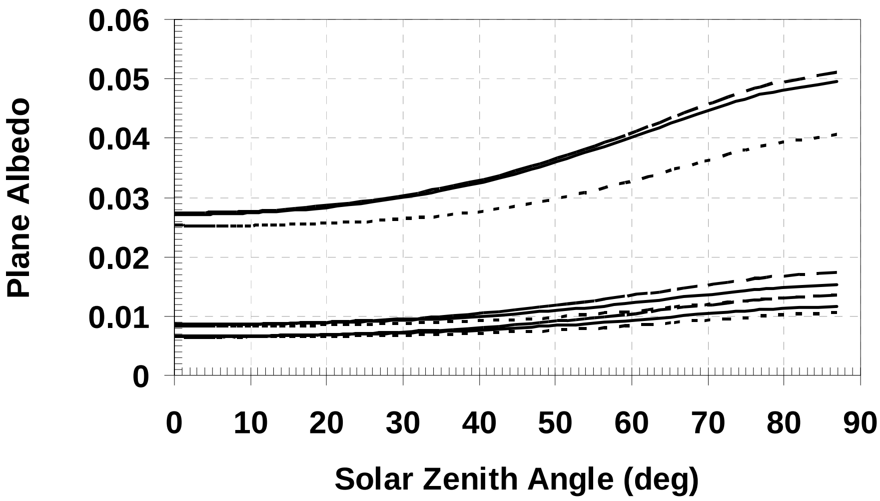

Fig. 4b. Plane albedo calculated using exact radiative transfer calculations (solid curves), QSSA, Eq. (5) (dotted curves) and our approximation, Eq. (7) (dashed curves) with g = 0.9583 and B = 0.008692 at ω_0 = 0.7, 0.75, 0.9 (from the bottom upwards).

Similar to the situation with RF, QSSA for the PA demonstrates a high accuracy for all clear waters, and even for turbid ones, up to $\omega_0 = 0.75$. At larger values of ω_0, it is better to use Eq. (7). For example, at $g = 0.9583$ and $\omega_0 > 0.9$, an error for the PA, calculated in the framework of QSSA can reach 20% and more (at $\theta_0 > 60^\circ$), while polynomial approximation, derived by us, yields maximum 15% error (Fig. 5b).

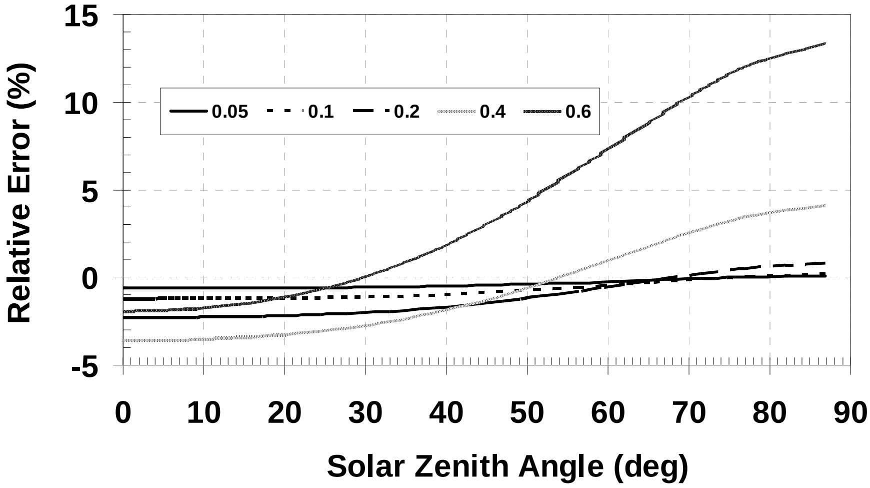

Fig. 5a. Relative errors of Eq. (5) as compared to exact radiative transfer calculations with $g = 0.5033$ and $B = 0.1559$ at $\omega_0 = 0.05, 0.1, 0.2, 0.4, 0.6$.

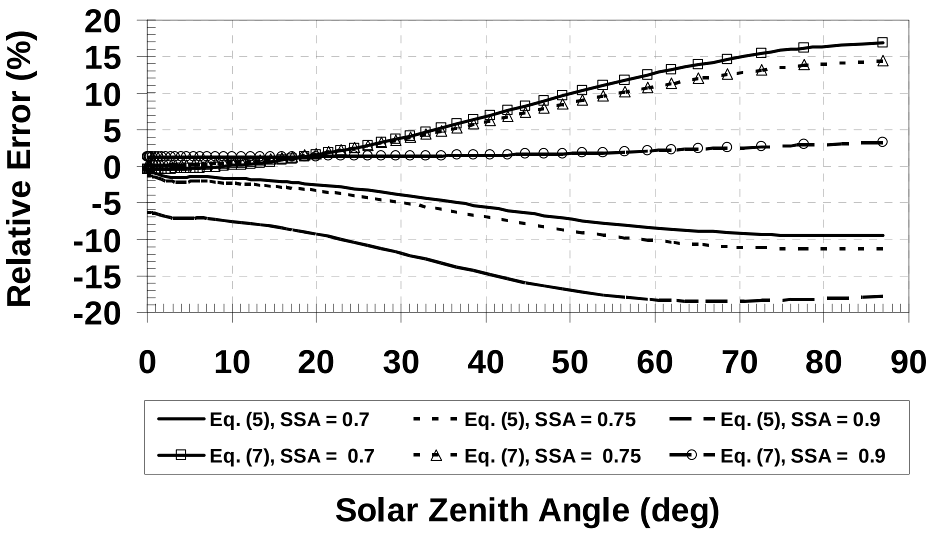

Fig. 5b. Relative errors of QSSA, Eq. (5) and Eq. (7) as compared to exact radiative transfer calculations with $g = 0.9583$ and $B = 0.008692$ at $\omega_0 = 0.7, 0.75, 0.9$.

SPHERICAL ALBEDO

The spherical albedo, due to angle averaging is a most easily modeled type of reflectance. There are a lot of approximations of this quantity, however, only a small portion of them yield acceptable results[4]. In the current study we represent three approximations: QSSA, Hulst's approximation (HA), and Madgett-Richards approximation (MRA). The expression for QSSA, applied to the spherical albedo, is derived by introduction of Eq. (5) to Eq. (2):

$$r(\omega_0, F) = \frac{\omega_0}{1-\omega_0 F} \sum_{i=0}^{N} \int_0^1 (-1)^i x_i P_i(\mu_1) Q_i(\mu_1) \mu_1 d\mu_1 \,, \; \mu_1 \equiv \cos\theta_1 \,, \qquad (11)$$

where relation of $P_i(\theta)$ with $p(\theta)$ and x_i for a given phase function defined by Eq. (4).

The HA is expressed as follows[6]:

$$r(g, \omega_0) = \frac{(1-s)(1-0.139s)}{1+1.17s}, \qquad (12)$$

where the similarity parameter s is defined by Eq. (8). The spherical albedo is calculated in the framework of the MRA approximation as follows[12]:

$$r(g, \omega_0) = 1 + \gamma - \sqrt{\gamma(2+\gamma)}\,, \; \gamma = 8(1-\omega_0)/3\omega_0(1-g)\,. \qquad (13)$$

The plots of SA calculated for two extreme natural conditions, described above, according to different approximations are presented in Fig. 6 and plots of errors derived by the comparison with exact calculations shown in Fig. 7. Like the situation with reflection function and plane albedo, QSSA yields better accuracy for spherical albedo of modeled clear waters, but it gives a significant error in the case of turbid waters with very high values of ω_0.

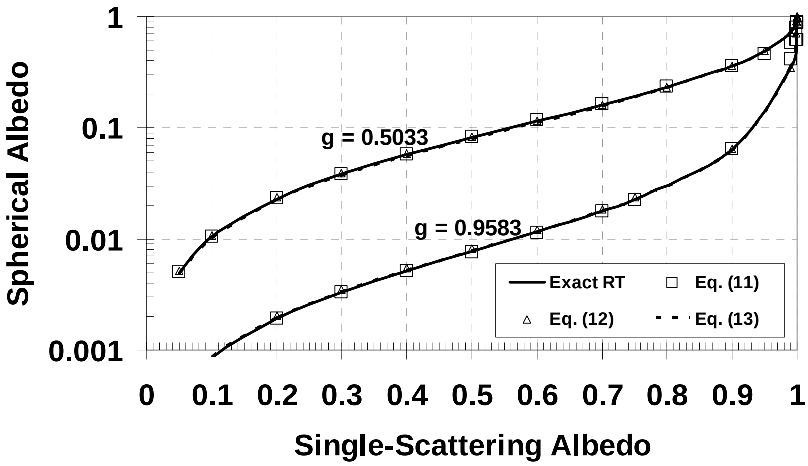

Fig. 6. Spherical albedo obtained using exact radiative transfer calculations and some approximations with *g* = 0.5033 and *g* = 0.9583.

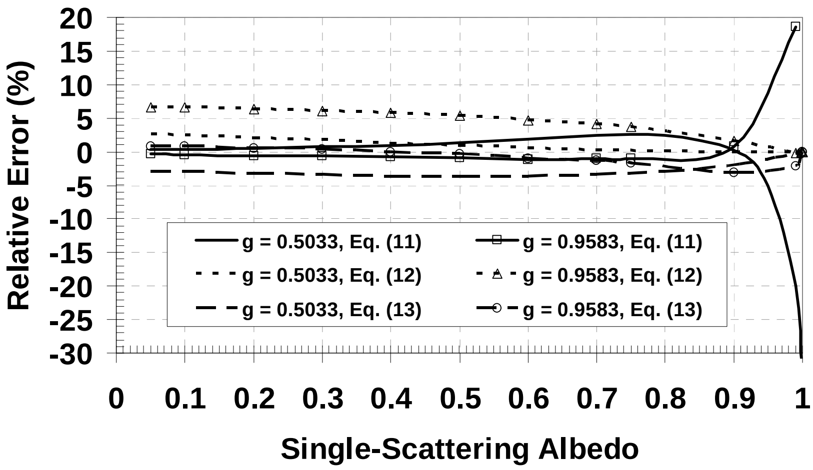

Fig. 7. Relative errors of QSSA, Eq. (11); HA, Eq. (12); and MRA, Eq. (13) as compared to exact radiative transfer calculations with *g* = 0.5033 and *g* = 0.9583.

DISCUSSION

Several approximations of reflective features (reflection function, plane albedo and spherical albedo) are compared with exact radiative transfer calculations for semi-infinite light scattering media. It is shown that the quasi-single-scattering approximation (QSSA) yields acceptable results in comparison with other approximations. For example, at single-scattering albedo $\omega_0 < 0.9$ and intermediate incidence angles, an accuracy of QSSA is better than 28% for the reflection function (Fig. 3b), 18% for the plane albedo (Fig. 5b) and 2.5% for the spherical albedo (Fig. 7). Nevertheless, some other very simple approximations as shown above often yield results more accurate than QSSA. We did not show here an impact of the phase functions on the accuracy of approximate formulae. It is known, however, that the error for reflection function is smaller, if the scattering phase functions are flat in the backward scattering hemisphere[13, 14, 15, 16]. Additional studies of the accuracy of QSSA approximation for various phase functions and values of ω_0 are given by Gordon[1-3], Golubitsky et al.[15], Golubitsky and Levin[16], and Zege[13]. Our own study show that for the more flat phase function (we checked out the Fournier-Forand phase function[17]) than it used in current study, all three characteristics have approximately the same errors as for considered here, with only one exception: for the small solar zenith angles (approximately less than 20 degrees) errors in reflection function become significantly lesser, about the same order as for the larger angles.

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