1.1

Lecture 1.1   Introduction

Maxwell-Boltzmann to Alder & Wainwright via long-tailed probability distributions and the scaling of atmospheric temperature and winds.

Key References

Alder & Wainwright (1970), *Phys. Rev. A,* **1,** 18-21.
[emergence of fluid flow from molecular dynamics]

Schertzer & Lovejoy (1987), *J. Geophys. Res.,* **92,** 9693-9714.
[generalized scale invariance, statistical multifractals]

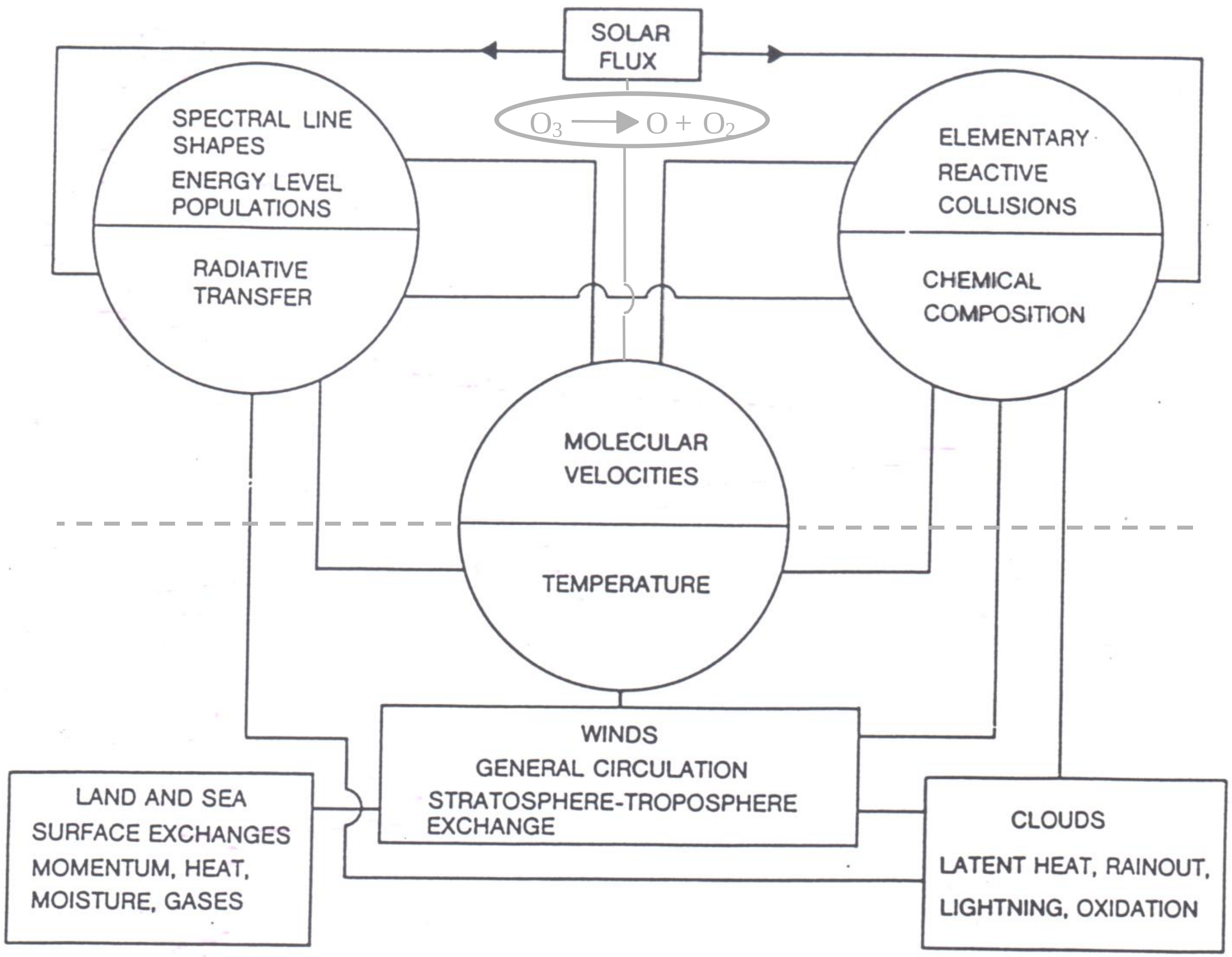
Tuck (2008), *Atmospheric Turbulence: A Molecular Dynamics Perspective*. Oxford University Press.

Tuck (2010, 2011), *Q. J. R. Meteorol. Soc.,* **136,** 1125-1144 & **137,** 275.

Schertzer & Lovejoy (2011), *Int. J. Bifurc. Chaos,* **21(12),** 3417-3456.
[most recent review of generalized scale invariance]

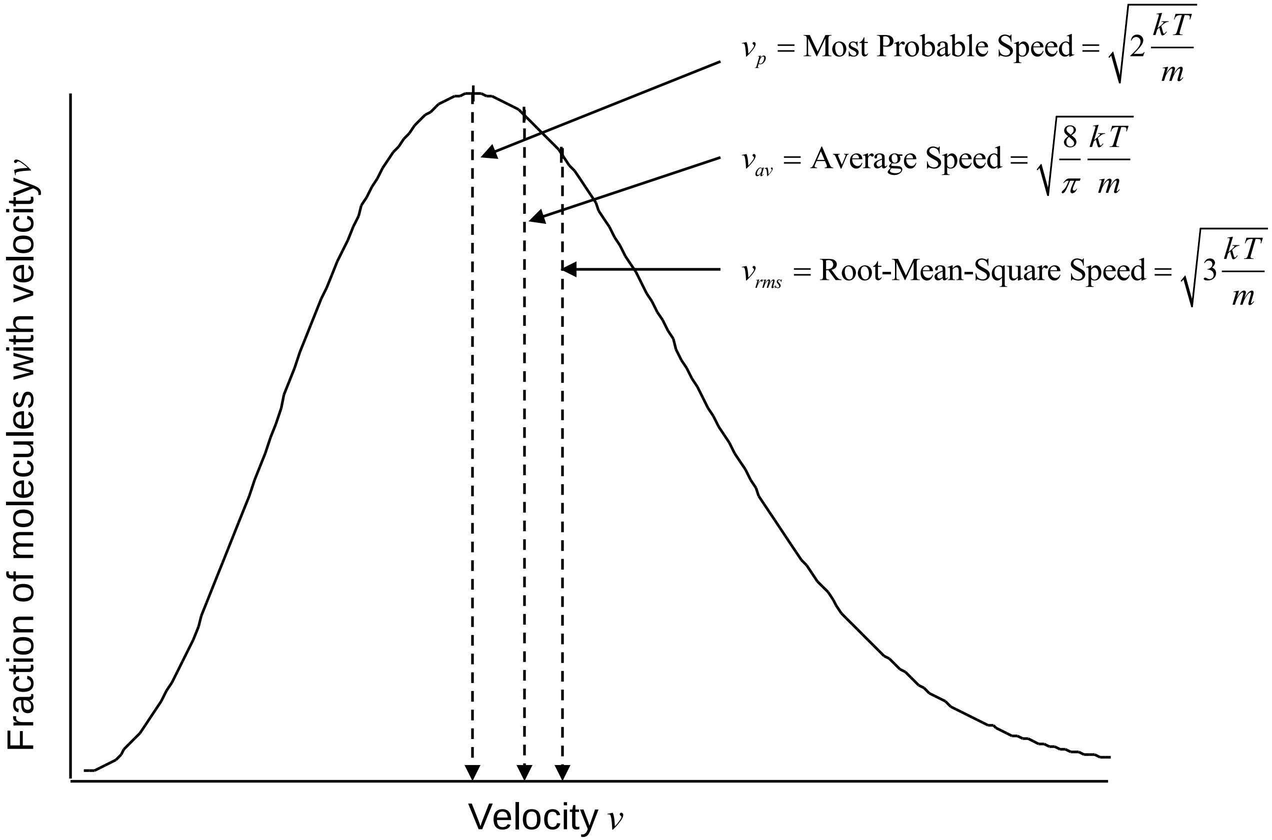
Lovejoy & Schertzer (2012), *The Weather and the Climate: Emergent Laws and Multifractal Cascades.* Cambridge University Press.

The correspondence and coupling of the microscopic and macroscopic processes in the atmosphere.

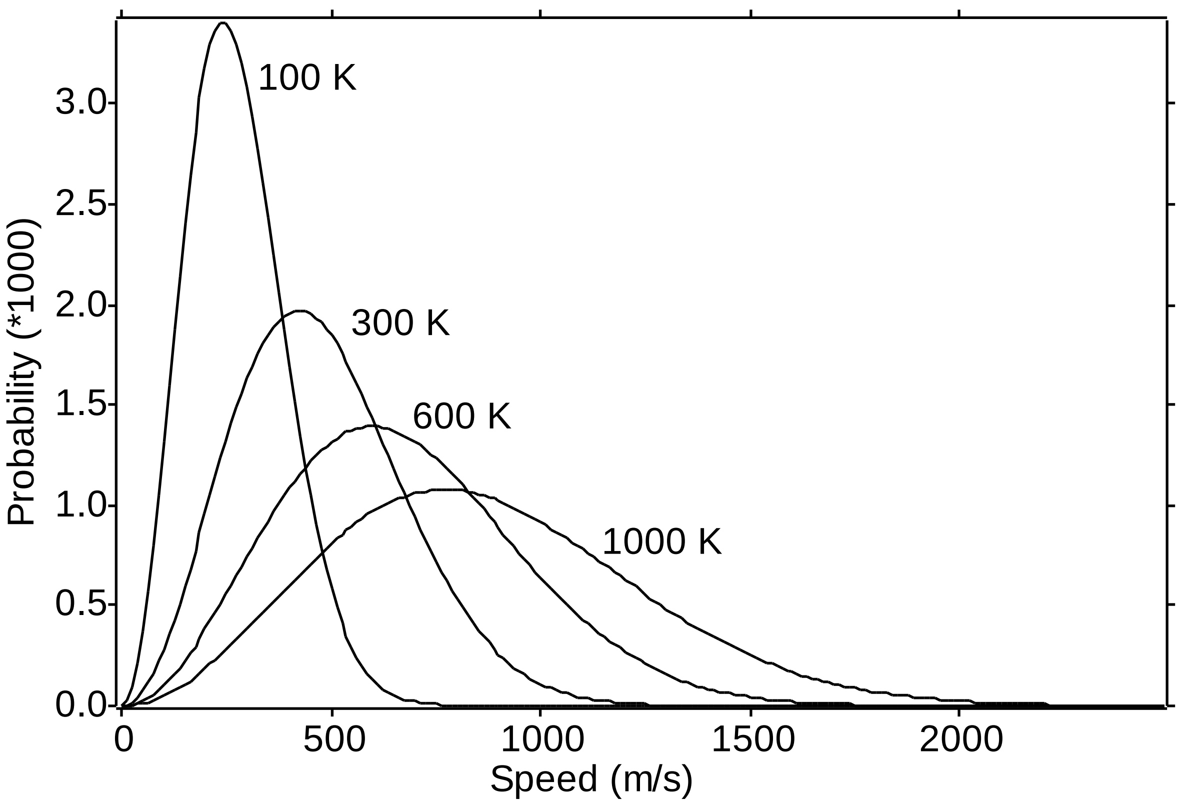


Maxwellian Velocity Distribution

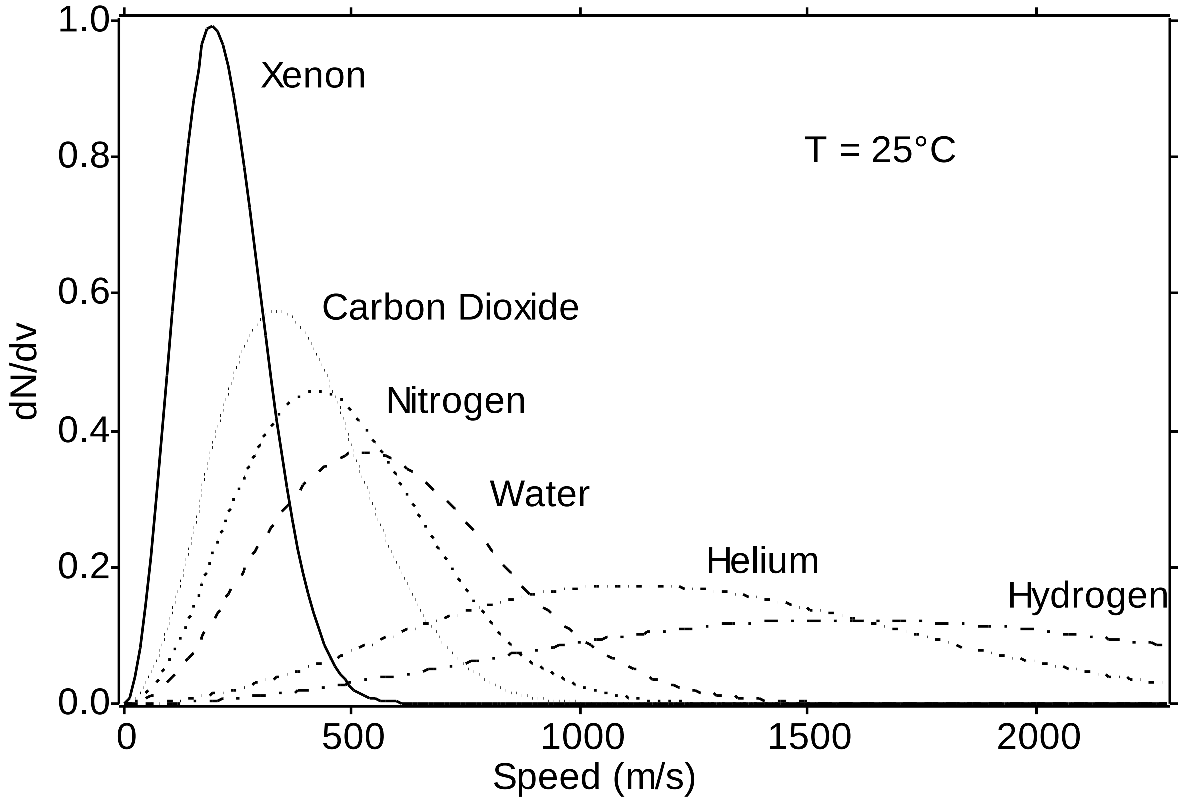
$$\frac{N_V(v)}{N} = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{KE}{kT}}$$



Maxwellian speed PDFs, *m* = 28: *T* dependence



Maxwellian speed PDFs: mass dependence



DEFINITION

Atmospheric temperature is what is measured by your thermometer (meteorological).

The absolute temperature, *T*, of a system is the reciprocal of the derivative of the entropy, *S*, with respect to its energy, *E* :

$$\frac{dS}{dE} = \frac{1}{kT}$$

[Landau & Lifshitz, (1980), *Statistical Physics, Course of Theoretical Physics, Vol. 5, 3rd ed., Chapters 1 - 3.]*

T is purely statistical, having strict meaning only for macroscopic bodies at equilibrium. One can of course observe with a calibrated thermometer. It averages over the velocity distribution of the molecules impinging upon it.

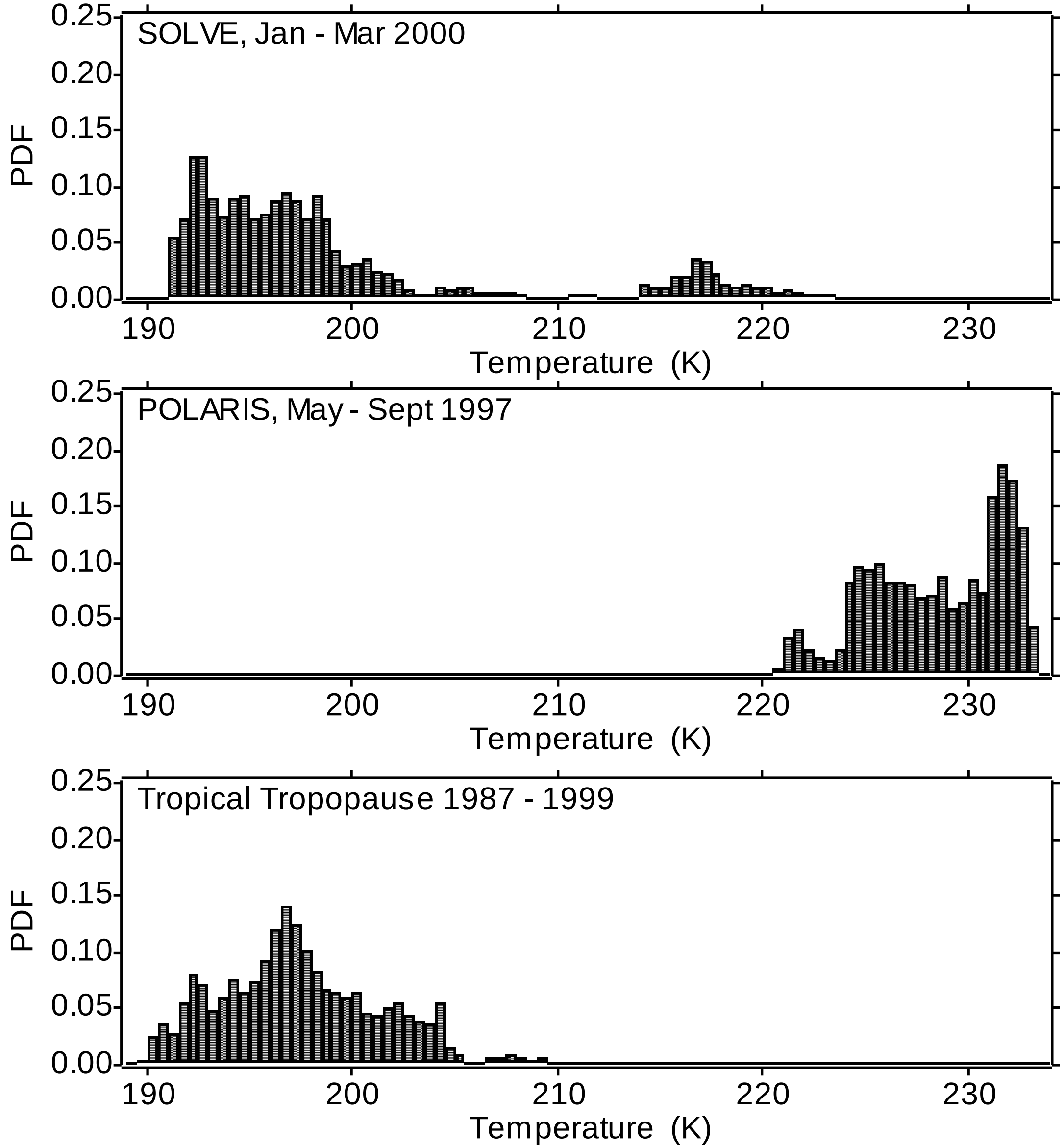
MICROSCOPIC VIEW OF TEMPERATURE

- The total kinetic energy of *N* classical particles is 3*N*k*T*/2.

- In terms of distributions of molecular velocity v

$$\overline{v_x^2} = \sqrt{\frac{m}{2\pi kT}} \int_{-\infty}^{\infty} v_x^2 e^{-m v_x^2 / 2kT} dv_x = \frac{kT}{m}$$

Long-tailed PDFs of temperature: millions of 5 Hz points from scores of ER-2 flight segments, Arctic summer 1997 & winter 2000, ER-2 & WB57F tropical tropopause 1987-1999.



Slide 11

Scale Invariance and Statistical Multifractality

$$S_q(r;\Psi) = \langle \left| \Psi(x+r) - \Psi(x) \right|^q \rangle$$

↑ q^{th} order structure function S of variable $\Psi(x)$

If a plot of $\log S_q$ vs. $\log(r)$ is linear with slope $\zeta(q)$, then $\zeta(q)$ is a scaling exponent for $\Psi(x)$, which therefore has scale invariance and power law PDFs.

Define

$$H(q) = \zeta(q)/q$$

Further define

$$H = H_q + K(q)/q$$

We will be interested in K(q) and H

To obtain ***K*(*q*)**, consider $\Psi(x)$ to have been observed at finite intervals $x = 1,2,3\ldots.. x_{max}$ and define:-

$$\varepsilon(1,x) = \{|\Psi(x+1) - \Psi(x)|\}/\langle\Psi(x+1) - \Psi(x)|\rangle$$

for $x = 1,2,3. \ldots.. x_{max}$

$$\varepsilon(r,x) = (1/r)\Sigma_{j=x}^{x+r-1}\varepsilon(1,j)$$

for $x = 1,2,3, \ldots.. x_{max} - r$

then a plot of $\log\langle\varepsilon(r,x)\rangle^q$ vs $\log r$ has slope $-K(q)$ and a plot of $-K(q)$ vs q shows a convex function with $K(0) = K(1) = 0$.

We can now note equivalences between scale invariance and statistical thermodynamics.

Formal equivalences between **scale invariant** (r.h.s.) and **statistical thermodynamic** (l.h.s.) variables

$T = 1/q k_{Boltzmann}$ **temperature**

$f = e^{-K(q)}$ **partition function**

$G = -K(q)/q$ **Gibbs free energy**

This offers possible links:

molecular scale
↓
statistical thermodynamics
↓
macroscopic scale invariant observables

Basic scaling relations for atmospheric turbulence

Consider horizontal velocity v with fluctuations Δv over horizontal length interval Δx and vertical interval Δz :

$$\Delta \boldsymbol{v} = \text{ø(horiz)}.(\Delta x)^{H(horiz)}$$

$$\Delta \boldsymbol{v} = \text{ø(vert)}.(\Delta z)^{H(vert)}$$

where ø(horiz) is the turbulent energy flux ε in the vertical and Ø(vert) is the turbulent energy flux; H(horiz) and H(vert) are the associated power law exponents. The Kolmogorov law for isotropic turbulence is obtained by setting

Ø(horiz) = ø(vert) = $\varepsilon^{1/3}$ and **H(horiz) = H(vert) = 1/3**

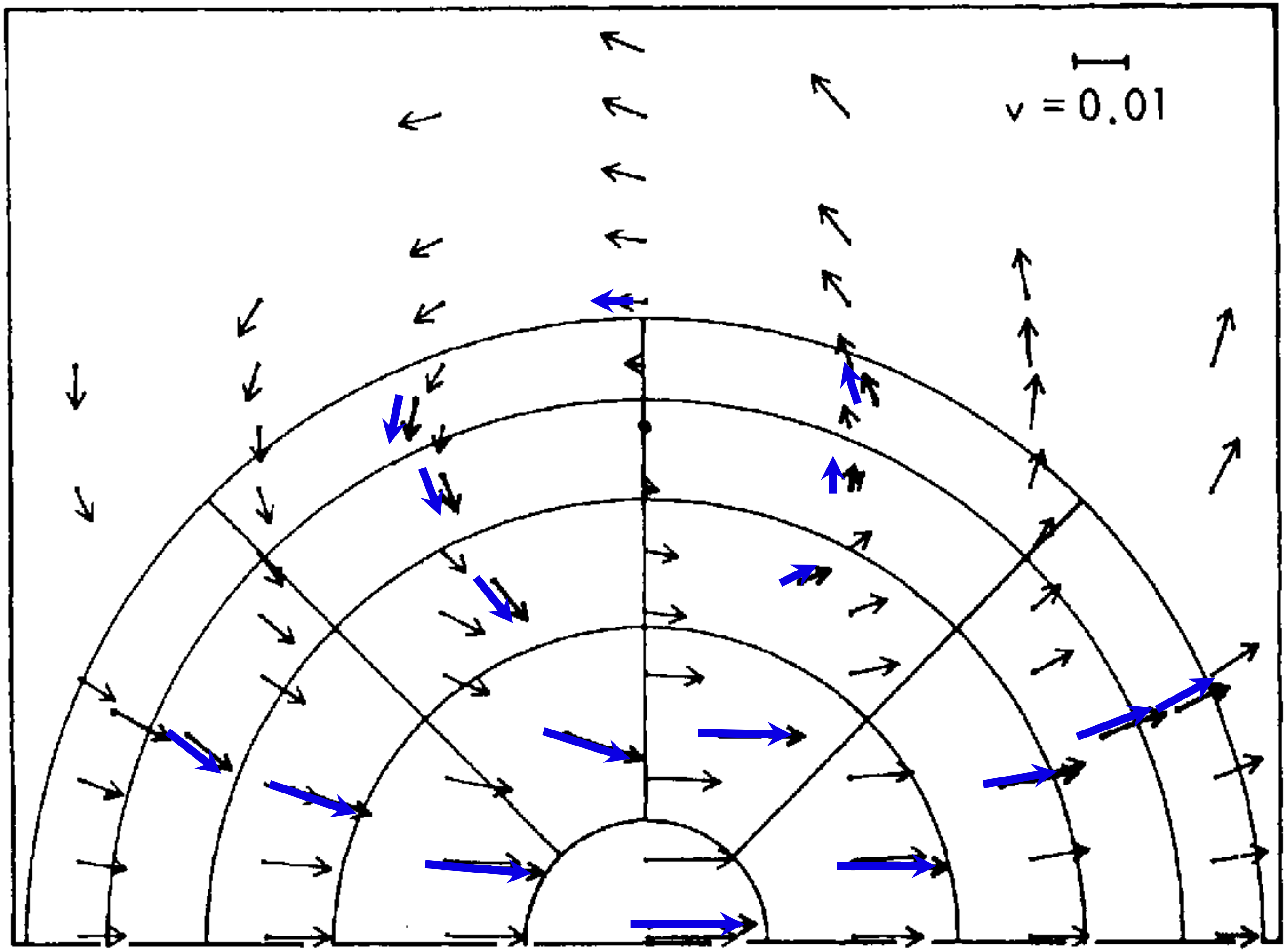
However, as we shall see later, this is at odds with observations. We therefore **assume** via **dimensional analysis** relations that preserve scaling while permitting anisotropy:

Ø(horiz) = $\varepsilon^{1/3}$ implying **H(horiz) = 1/3**

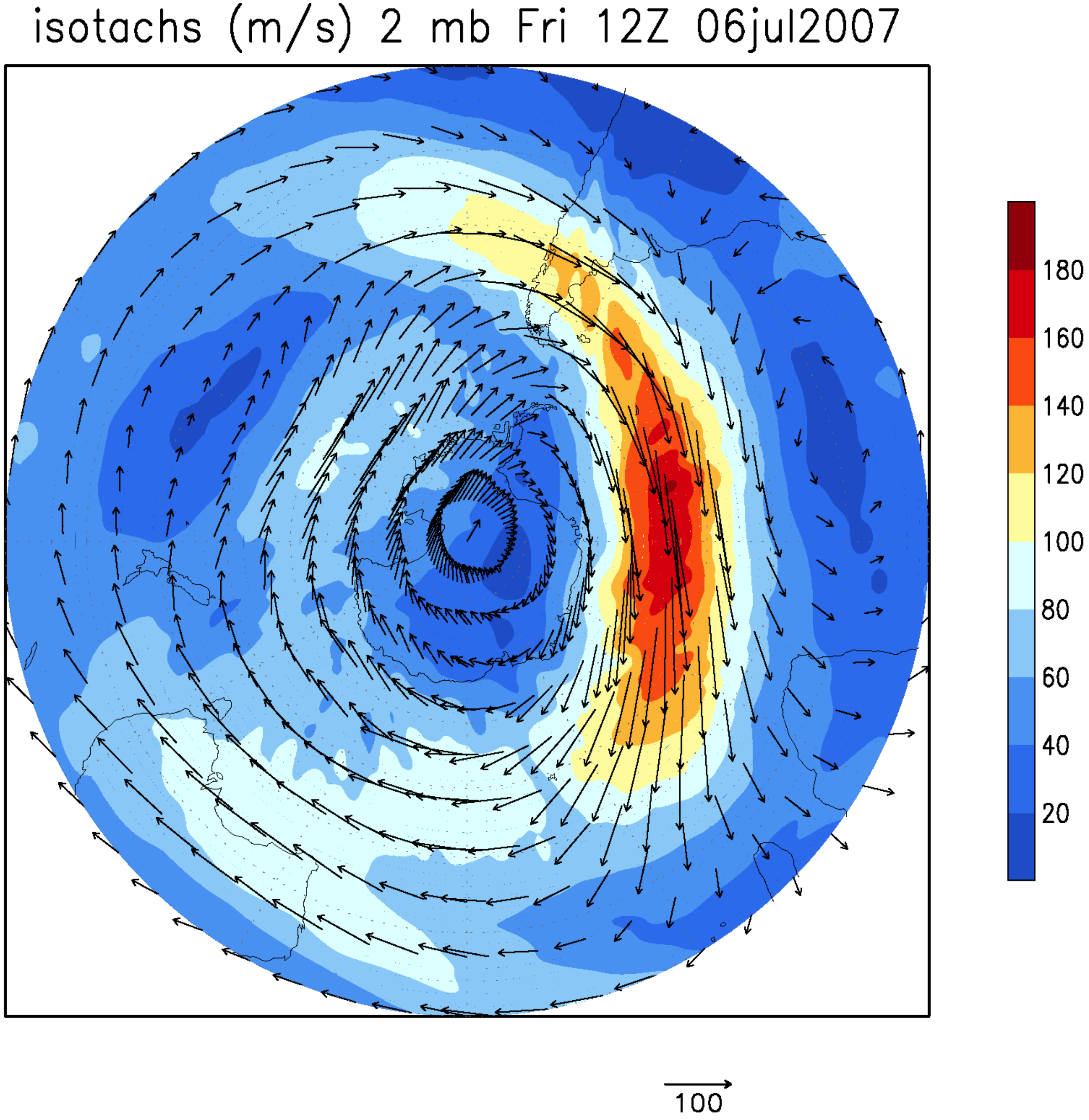
Ø(vert) = $\eta^{1/5}$ implying **H(vert) = 3/5**

where energy flux ε (m^2s^{-3}) dominates in the horizontal and buoyancy variance flux η (m^2s^{-5}) dominates in the vertical.

Alder & Wainwright (1970): molecular dynamics simulation of a flux applied to an equilibrated Maxwellian population results in the emergence of vortices on scales of 10^{-12} seconds & 10^{-8} metres.



NOAA-NCEP GFS 0.5^0 RESOLUTION ANALYSIS, FLECHES & ISOTACHS



1.1 Summary

• Organized flow emerges from a randomized (thermal) population of Maxwellian ‘billiard balls’ subject to an anisotropic flux simulated by molecular dynamics, on very short time and space scales.

•Observed temperature distributions in the atmosphere are not of Maxwell-Boltzmann character.

•To accommodate the observed non-M-B PDFs, we must lift the assumption of isotropy in the atmosphere.

•The atmosphere has permanent anisotropies: gravity, planetary rotation, solar beam, planetary surface and, for any one scale except the great circle, larger scale winds.

•Abandoning isotropic motions on any scale yields predictions:

H(horiz) = 1/3
H(vert) = 3/5

and so H(horiz)/H(vert) = 5/9

*How do these work out in practice? See next lecture ……