2.2
Lecture 2.2  Vertical Scaling

‘Vertical’ observations from GPS dropsondes released by the NOAA Gulfstream 4 aircraft, and ascents and descents from the G4 and the NASA WB57F.

Recall theoretical prediction: $H_{\text{vertical}} = 3/5$
### Dropsonde

**GPS Dropwindsonde Specifications**

<table>
<thead>
<tr>
<th></th>
<th>Operate</th>
<th>Accuracy</th>
<th>Resolution</th>
<th>Time Const.</th>
<th>Typ. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>1060-20mb</td>
<td>0.5mb</td>
<td>0.1mb</td>
<td>&lt;0.01 sec</td>
<td>1.0mb</td>
</tr>
<tr>
<td>Temperature</td>
<td>-90 to 45°C</td>
<td>0.2°C</td>
<td>0.1°C</td>
<td>2.5 to 3.7 s</td>
<td>0.2°C</td>
</tr>
<tr>
<td>Humidity</td>
<td>0 to 100%</td>
<td>2.0%</td>
<td>0.1%</td>
<td>0.1 to 10 sec</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Winds</td>
<td>0 to 150 m/s</td>
<td>0.5 m/s</td>
<td>0.1 m/s</td>
<td>0.5 to 2.0 ms</td>
<td></td>
</tr>
</tbody>
</table>

- **NCAR GPS Dropsonde**
  - the definitive atmospheric profiling tool
  - Square-cone Parachute
  - Vents fill chute within 10 seconds after release from aircraft
  - Increased stability of dropsonde
  - GPS Antenna
  - Microprocessor controls the transmitter and digitizes data from the sensors
  - Battery pack provides power for at least one hour
  - GPS Receiver collects the data from GPS satellites used to calculate wind speed and direction
  - Humidity sensors and temperature sensor
  - Radio Transmitter sends temperature, humidity, pressure, and GPS (wind) data to the aircraft every 0.5 seconds
  - Sondie Dimensions: Length 16" Diameter 2.75" Weight: 0.85 lbs
  - Fall Speed ranges from 36 mph at 20,000 feet to 24 mph at sea level. A drop from 20,000 feet lasts 7 minutes.

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Winter Storms 2004229 - 20040314

Numbers are dates (MMDD)

Longitude (°E)
-170 -160 -150 -140 -130 -120

Latitude (°N)
60 50 40 30 20

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Dropsondes: The vertical resolution is multifractal

Mean speed decreases from about \(\approx 18 \text{m/s (12km)}\) to \(\approx 9 \text{ m/s (surface)}\)

Minimum sampling frequency: 0.5 s
\(\approx 5 \text{ m}\)

\[ \left\langle |\Delta z|^q \right\rangle \approx \Delta t^{-\varphi(q)} \]
Blow up of intermittent lower layer

Well reproduced from both sondes
Dynamic stability: marine boundary layer, troposphere
Potential Temperature, $\theta$

- $\theta$ is an alternative vertical coordinate in meteorology.

- It is the temperature an air element would have if it was brought adiabatically to the surface.

\[
S = C_p \ln \theta + \text{constant}
\]

where $S$ is entropy
Generalised Scale Invariance: Exponents

- $H$ Hurst or conservation exponent ($0<H<1$)
- $C_1$ intermittency exponent ($0<C_1<1$)
- $\alpha$ multifractality exponent ($0<\alpha<2$)

*Atmosphere has $H = 0.56$, $C_1 = 0.05$, $\alpha = 1.6$ from horizontal aircraft observations.

*Gaussian has $H = 0.5$, $C_1 = 0$, $\alpha = 2$

*What of the vertical scaling?
The figures to the right are composite variograms, created by overlaying the individual variograms computed for each dropsonde and then fitting a line to the aggregate.

While variograms typically involve variance, we use the first order structure function in order to minimize intermittency corrections and to facilitate comparison with theoretical (dimensional analysis) exponents.

Each individual variogram contained about 100 points, and there were 235 drops that successfully measured wind speed, and 246 that measured temperature and relative humidity. Therefore the lines to the right are each fitting roughly 24,000 points. The errors are 95% confidence intervals.

The surprise is that the slope (i.e. \( H \)) for horizontal wind speed, came out appreciably higher than the Bolgiano-Obukhov theoretical value of 0.6. This indicates smoother than expected horizontal wind speed profiles. It is clear also that temperature behaves differently in the vertical than the other variables.

Subsequent spectral analysis has shown that the near-unity value of \( H \) for temperature is an artifact of the structure function method, which does not produce a good estimate of \( H \) when \( H > 1 \) or \( H < 0 \). For the data of 20040229, the spectral method yielded \( H \approx 5/4 \), again a value unique to temperature.
Vertical scaling of horizontal wind, 235 dropsondes, Winter Storms 2004. Scaling is not Kolmogorov or gravity wave; Bolgiano-Obukhov is close in boundary layer, but none are correct at jet altitudes.
WS06 Wind Speed
Dynamical stability $[Ri>0.25]$ at 500 & 150 m(left), 50 & 10 m(right) Dropsonde (25°N,157°W) on 20040229. The ‘Russian doll’ structure.
Histogram of $H_v$ for 1 km layers
Velocity scaling exponents $H, \beta, C_1$ and $\alpha$
Lapse rate PDFs at different vertical resolutions, dropsondes from NOAA G4, 20040229 - 20040315, eastern Pacific Ocean.

(a)

Winter Storms 2004
Scale = 15 m

(b)

Winter Storms 2004
Scale = 100 m

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Winter Storms 2004
Scale = 5 m

PDF

Winter Storms 2004
Scale = 5 m

Altitude (m)

Winter Storms 2004
Scale = 5 m

Lapse Rate (K/m)
Winter Storms 2004
Scale = 400 m

PDF

K/m

Winter Storms 2004
Scale = 400 m

Altitude (m)

Lapse Rate (K/m)

PDF

12x10³

0.0

0.5

0.0

-0.5

-1.0

-0.05

0.0

0.5

1.0

-0.05

0.0

0.5

1.0

-1.0

-0.5

0.0

0.5

1.0

-0.05

0.0

0.5

1.0
Aircraft ascents and descents, Jan-Mar 2004, 10° - 60° N, 84° - 158° W

Gulfstream Ascents & Descents

Temperature

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Wind Speed

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Relative Humidity

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Slope = 0.95 ± 0.01

Slope = 0.68 ± 0.02

Slope = 0.66 ± 0.03

WB57F Ascents & Descents

Temperature

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Wind Speed

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Relative Humidity

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

Slope = 0.943 ± 0.023

Slope = 0.364 ± 0.037

Slope = 0.640 ± 0.039
Variogram, all 261 dropsondes during the mission

(a) Temperature

(b) Wind Speed

(c) Relative Humidity

Slope = 0.986 ± 0.002

Slope = 0.768 ± 0.005

Slope = 0.750 ± 0.006
## Scaling from G4 in Winter Storms 2004

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dropsondes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>$0.986 \pm 0.002$</td>
<td>$0.95 \pm 0.02$</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>$0.768 \pm 0.005$</td>
<td>$0.68 \pm 0.02$</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>$0.750 \pm 0.006$</td>
<td>$0.66 \pm 0.03$</td>
</tr>
</tbody>
</table>

*Vertical & horizontal exponents are different; no isotropy!*

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Co-dimensions of vertical stability criteria

<table>
<thead>
<tr>
<th>CRITERION</th>
<th>CO-DIMENSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^2 (\theta) = g \partial \log \theta / \partial z$</td>
<td>$0.36 \pm 0.06$</td>
</tr>
<tr>
<td>Brunt-Väisälä (dry)</td>
<td></td>
</tr>
<tr>
<td>$Ri = N^2 (\theta) [\partial v / \partial z]^{-2}$</td>
<td>$0.22 \pm 0.04$</td>
</tr>
<tr>
<td>Richardson Number</td>
<td></td>
</tr>
<tr>
<td>$N^2 (\theta_w) = \partial \theta_w / \partial z$</td>
<td>$0.15 \pm 0.02$</td>
</tr>
<tr>
<td>Moist static stability</td>
<td></td>
</tr>
</tbody>
</table>

Water - via its latent heat entropy - makes a big difference to atmospheric stability on all scales.
Lecture 2.2 Summary

* GPS dropsondes are very effective at observing the vertical structure of wind, temperature and humidity.

* Variables do not follow Gaussian statistics. They show statistical multifractal scale invariance.
  * No monolithic stable layers; no isotropic turbulence.
  * Jet streams alter scaling exponent $H$, horiz & vert.

* Atmospheric moisture greatly affects vertical scaling.

* Vertical scaling of temperature different than in horizontal.