3.1
Lecture 3.1 Horizontal Scaling

Derived from aircraft observations - a more complicated process than it first appeared to be.
We have used 4 different research aircraft:

- **NASA ER-2**, flown at constant Mach number $M$ where

$$M = \frac{v_{\text{aircraft}}}{\gamma RT}^{0.5}$$

- **NASA WB57F**, flown either at constant $M$ or isobarically at constant pressure $P$

- **NOAA Gulfstream 4**, flown at constant $P$

- **NASA DC-8**, flown at constant $P$

- All the aircraft can be flown manually, something that is immediately obvious when it happens - the scaling is destroyed.

* With knowledge of the vertical scaling, it becomes apparent that the manner of flight (autopilot) matters in extracting the horizontal scaling. (The ideal does not exist:- control to constant geometric altitude).
NASA ER-2
ER-2 flight into the polar night jet, Stavanger to Wallops Is.: $T$

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ER-2 flight into the polar night jet, Stavanger to Wallops Is.: $\sqrt{(u^2+v^2)}$
19890220 Wind Vectors
Centred At (55° N, 46° W)

8192 sec

1024 sec

128 sec
19890220 Wind Shear Vectors

8192 sec

1024 sec

128 sec
Scaling Calculation for 19890220 Wind Speed

\[ \log(\langle|f(x+r) - f(x)|\rangle) \]

\[ \log( r ) \]

\[ H_z = 0.45 \pm 0.04 \]
Scaling Calculation for 19941005 Wind Speed 
(44° S, 173° E) to (65° S, 180° E)

\[ H_z = 0.70 \pm 0.06 \]
All ER-2 flight segments, 1987 - 2000, 90°N - 72°S

ER-2 Temperature

\[ \log( |f(x+r) - f(x)|) \]

Slope = 0.531 ± 0.007

ER-2 Wind Speed

\[ \log( |f(x+r) - f(x)|) \]

Slope = 0.509 ± 0.007
Aircraft ascents and descents, Jan-Mar 2004, 10° - 60° N, 84° - 158° W

Gulfstream Ascents & Descents

Temperature

Slope = 0.95 ± 0.01

Wind Speed

Slope = 0.68 ± 0.02

Relative Humidity

Slope = 0.66 ± 0.03

WB57F Ascents & Descents

Temperature

Slope = 0.94 ± 0.023

Wind Speed

Slope = 0.364 ± 0.037

Relative Humidity

Slope = 0.64 ± 0.039

Gulfstream Ascents & Descents

WB57F Ascents & Descents

Aircraft ascents and descents, Jan-Mar 2004, 10° - 60° N, 84° - 158° W

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All 261 dropsondes, Winter Storms 2004, 10°N-46°N, 140°W-172°W, 20040229 - 20040314, NOAA G4

Temperature
Slope = 0.986 ± 0.002

Wind Speed
Slope = 0.768 ± 0.005
All 261 dropsondes, Winter Storms 2004, 10°N-46°N, 140°W-172°W, 20040229 - 20040314, NOAA G4

Relative Humidity

Slope = 0.750 ± 0.006
Scaling from G4 in Winter Storms 2004

N.B. Temperature scales differently than wind speed and humidity

<table>
<thead>
<tr>
<th>Dropsondes</th>
<th>‘Vertical’ Aircraft Segments</th>
<th>‘Horizontal’ Aircraft Segments</th>
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<tbody>
<tr>
<td>Temperature</td>
<td>0.986 ± 0.002</td>
<td>0.95 ± 0.02</td>
</tr>
<tr>
<td>Wind Speed</td>
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</table>
All WB57F ‘horizontal’ data near tropical tropopause 1998 - 1999 (WAM and ACCENT)

**WB57 Temperature**

- Slope = 0.585 ± 0.023

**WB57 Wind Speed**

- Slope = 0.507 ± 0.028
All DC-8 ‘horizontal’ data, 44°S - 90°S, Aug-Sep 1987 (AAOE)

DC8 Temperature

Slope = 0.620 ± 0.017

DC8 Wind Speed

Slope = 0.553 ± 0.013
All DC-8 total water, ‘horizontal’, 44°S - 90°S, Aug-Sep 1987

DC8 Water

Slope = 0.271 ± 0.033
All NOAA G4 ‘horizontal’ data, 10°N-46°N, 140°W-172°W
20040229 - 20040315

Temperature

Wind Speed

log( <|f(x+r) - f(x)|> )

log( r )

Slope = 0.52 ± 0.02

Slope = 0.56 ± 0.02
Highest quality data, all TAMDAR flights over continental USA in 2009 between 5.0 and 5.5 km, about 14,500 flights, single aircraft only, 20 m deep (1.26 hPa) sections.

* $v$ is wind speed, $x$ is horizontal displacement, $z$ is vertical displacement.

* Increments are squared, averaged and plotted below, transverse (top), longitudinal (bottom).

* Black is data, purple generalized scale invariance theory.

* Yields $H = 0.57$, close to GSI theory of 5/9.
All ER-2 ozone & nitrous oxide, 59°N-70°S, heavy SH weighting

\[ \log(\langle |f(x+r) - f(x)| \rangle) \]

\[ \log( r ) \]

Slope = 0.49 ± 0.01

Slope = 0.56 ± 0.01
Summary, Lecture 3.1

* Vertical and horizontal scaling are different.

* Aircraft trajectories are fractal, and mix horizontal and vertical scaling. If this is not allowed for by careful analysis of accurate, single aircraft data, illusory scale breaks appear.

* There is an altitude dependence in the vertical scaling of the horizontal wind, for which no theoretical explanation currently exists.

* The value of $0.56 \pm 0.01$ for $H(\text{N}_2\text{O})$ is very strong evidence for the $H = 5/9$ scaling theory, since this tracer does not affect the trajectory of the aircraft.