4.1
Lecture 4.1, Consequences [1] of observed scale invariance

* No isotropic turbulence or diffusion, no normal distributions, no stable layers, means converge but variances do not, no local thermodynamic equilibrium, molecular properties must be accounted for by means other than the gas constant.

* These points imply difficulties in the formulation of numerical models. An example is given.

* Question: stratospheric polar vortex - containment vessel or flow reactor?
WB57F, Rocky Mountains 19980411. Severe turbulence & lee waves. Isentropes observed by MTP.
Lee waves near Riverton, Wyoming. Severe turbulence (WAM).
Scaling of WB57F observations and of MM5 simulation, 19980411.
Scaling of WB57F and MM5, WAM Rocky Mountain Lee Wave Flight 19980411

<table>
<thead>
<tr>
<th>Data set</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$H_6$</th>
<th>$\beta$</th>
<th>$2H_2 + 1$</th>
<th>$C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTW Temperature</td>
<td>0.36</td>
<td>0.33</td>
<td>0.23</td>
<td>1.59</td>
<td>1.66</td>
<td>0.05</td>
</tr>
<tr>
<td>MM5 Temperature</td>
<td>0.64</td>
<td>0.58</td>
<td>0.50</td>
<td>2.00</td>
<td>2.16</td>
<td>0.09</td>
</tr>
<tr>
<td>INS Wind Speed</td>
<td>0.29</td>
<td>0.25</td>
<td>0.15</td>
<td>1.83</td>
<td>1.5</td>
<td>0.05</td>
</tr>
<tr>
<td>MM5 Wind Speed</td>
<td>0.58</td>
<td>0.53</td>
<td>0.45</td>
<td>2.05</td>
<td>2.06</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Simulated monofractal signals: random, antipersistent & persistent.
Simulated statistical multifractal signal, with typical observed values of generalized scale invariance exponents: conservation, intermittency and Lévy.

Although using typical exponents, the trace still does not “look like” actual observed ones.
Intermittency, sharp gradients

19870823 ER-2 endascent-begindip

Wind Direction

Wind Speed

Horizontal Wind Speed (m/s)

Time (sec UTC)

Horizontal Wind Direction (deg CW from N)
Intermittency, sharp gradients

Horizontal Wind Speed (m/s)

Horizontal Wind Direction (deg CW from N)

19870909 ER-2 endascent-begindip

Slide 157
Intermittency, sharp gradients

Nitrous Oxide (ppbv)

Time (sec UTC)

Ozone (ppbv)

19870922 ER-2 endascent-begindip

Intermittency, sharp gradients
We now consider implications for a particular question: the degree of exchange across the edge of the stratospheric winter polar vortex, and the rates of reaction inside it.

* The scale invariance of the winds

* The scale invariance of some of the chemical species

* Application of the law of mass action
Scaling Exponents for Wind Speed

$H_a(S) = 0.45 \pm 0.04$

$H_a(S) = 0.70 \pm 0.06$
Scaling exponents $H_1$, $C_1$ and $\alpha$ for SOLVE ER-2, full precision at 5 Hz

<table>
<thead>
<tr>
<th>Date (yyymmdd)</th>
<th>Start &amp; stop times</th>
<th>$H_d$+$\Delta$+ci.</th>
<th>$C_d$+$\Delta$+ci.</th>
<th>$\alpha$+$\Delta$+ci.</th>
<th>$H_c$+$\Delta$+ci.</th>
<th>$C_c$+$\Delta$+ci.</th>
<th>$\alpha$+$\Delta$+ci.</th>
<th>$H_d$($O_1$)+ci.</th>
<th>$C_c$($O_1$)+ci.</th>
<th>$\alpha$($O_1$)+ci.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000111*</td>
<td>50118–54288</td>
<td>0.45 ± 0.05</td>
<td>0.042 ± 0.002</td>
<td>1.45 ± 0.33</td>
<td>0.40 ± 0.10</td>
<td>0.088 ± 0.006</td>
<td>1.34 ± 0.94</td>
<td>0.39 ± 0.06</td>
<td>0.070 ± 0.002</td>
<td>1.80 ± 0.15</td>
</tr>
<tr>
<td>20000111*</td>
<td>54288–59538</td>
<td>0.43 ± 0.05</td>
<td>0.060 ± 0.001</td>
<td>2.31 ± 0.74</td>
<td>0.39 ± 0.08</td>
<td>0.102 ± 0.004</td>
<td>1.48 ± 0.84</td>
<td>0.44 ± 0.05</td>
<td>0.052 ± 0.002</td>
<td>1.89 ± 0.15</td>
</tr>
<tr>
<td>20000114</td>
<td>46800–63931</td>
<td>0.52 ± 0.04</td>
<td>0.034 ± 0.001</td>
<td>1.72 ± 0.14</td>
<td>0.59 ± 0.05</td>
<td>0.067 ± 0.004</td>
<td>1.48 ± 0.56</td>
<td>0.44 ± 0.06</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>20000120</td>
<td>37553–47828</td>
<td>0.47 ± 0.06</td>
<td>0.026 ± 0.002</td>
<td>1.53 ± 0.21</td>
<td>0.48 ± 0.06</td>
<td>0.094 ± 0.007</td>
<td>1.78 ± 0.96</td>
<td>0.31 ± 0.04</td>
<td>0.023 ± 0.002</td>
<td>1.56 ± 0.15</td>
</tr>
<tr>
<td>20000123*</td>
<td>31017–38648</td>
<td>0.41 ± 0.05</td>
<td>0.027 ± 0.002</td>
<td>2.22 ± 1.39</td>
<td>0.57 ± 0.05</td>
<td>0.096 ± 0.007</td>
<td>1.84 ± 1.02</td>
<td>0.31 ± 0.06</td>
<td>0.023 ± 0.002</td>
<td>2.21 ± 0.92</td>
</tr>
<tr>
<td>20000127</td>
<td>45647–52267</td>
<td>0.43 ± 0.07</td>
<td>0.044 ± 0.002</td>
<td>2.38 ± 1.30</td>
<td>0.44 ± 0.07</td>
<td>0.085 ± 0.005</td>
<td>1.91 ± 1.97</td>
<td>0.41 ± 0.05</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>20000131</td>
<td>38199–42764</td>
<td>0.54 ± 0.05</td>
<td>0.037 ± 0.002</td>
<td>1.39 ± 0.24</td>
<td>0.54 ± 0.04</td>
<td>0.114 ± 0.008</td>
<td>1.83 ± 1.77</td>
<td>0.37 ± 0.07</td>
<td>0.027 ± 0.002</td>
<td>1.54 ± 0.15</td>
</tr>
<tr>
<td>20000202</td>
<td>42000–53500</td>
<td>0.54 ± 0.04</td>
<td>0.026 ± 0.002</td>
<td>1.37 ± 0.16</td>
<td>0.51 ± 0.05</td>
<td>0.094 ± 0.007</td>
<td>1.72 ± 1.85</td>
<td>0.41 ± 0.06</td>
<td>0.023 ± 0.002</td>
<td>1.39 ± 0.13</td>
</tr>
<tr>
<td>20000203</td>
<td>69353–72748</td>
<td>0.39 ± 0.04</td>
<td>0.038 ± 0.003</td>
<td>1.46 ± 0.22</td>
<td>0.31 ± 0.10</td>
<td>0.064 ± 0.005</td>
<td>1.11 ± 0.33</td>
<td>0.38 ± 0.08</td>
<td>0.037 ± 0.002</td>
<td>1.54 ± 0.25</td>
</tr>
<tr>
<td>20000226</td>
<td>31000–43000</td>
<td>0.57 ± 0.05</td>
<td>0.034 ± 0.002</td>
<td>1.38 ± 0.15</td>
<td>0.49 ± 0.05</td>
<td>0.076 ± 0.006</td>
<td>2.75 ± 2.00</td>
<td>0.39 ± 0.02</td>
<td>0.024 ± 0.002</td>
<td>1.53 ± 0.19</td>
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<tr>
<td>20000305</td>
<td>41747–52392</td>
<td>0.52 ± 0.03</td>
<td>0.032 ± 0.002</td>
<td>2.12 ± 1.23</td>
<td>0.55 ± 0.05</td>
<td>0.090 ± 0.006</td>
<td>2.30 ± 1.82</td>
<td>0.35 ± 0.06</td>
<td>0.026 ± 0.002</td>
<td>1.78 ± 0.21</td>
</tr>
<tr>
<td>20000307</td>
<td>37000–43000</td>
<td>0.46 ± 0.04</td>
<td>0.033 ± 0.002</td>
<td>1.43 ± 0.20</td>
<td>0.51 ± 0.05</td>
<td>0.104 ± 0.008</td>
<td>1.60 ± 1.00</td>
<td>0.26 ± 0.02</td>
<td>0.025 ± 0.002</td>
<td>1.33 ± 0.13</td>
</tr>
<tr>
<td>20000311</td>
<td>31524–39509</td>
<td>0.56 ± 0.06</td>
<td>0.035 ± 0.002</td>
<td>1.71 ± 0.63</td>
<td>0.60 ± 0.04</td>
<td>0.101 ± 0.008</td>
<td>1.59 ± 1.91</td>
<td>0.55 ± 0.04</td>
<td>0.042 ± 0.002</td>
<td>1.94 ± 0.23</td>
</tr>
<tr>
<td>20000311</td>
<td>42354–52389</td>
<td>0.62 ± 0.06</td>
<td>0.031 ± 0.002</td>
<td>1.55 ± 0.20</td>
<td>0.46 ± 0.05</td>
<td>0.093 ± 0.006</td>
<td>1.92 ± 1.71</td>
<td>0.55 ± 0.02</td>
<td>0.034 ± 0.002</td>
<td>1.82 ± 0.18</td>
</tr>
<tr>
<td>20000312</td>
<td>51934–58349</td>
<td>0.56 ± 0.04</td>
<td>0.030 ± 0.002</td>
<td>1.56 ± 0.18</td>
<td>0.44 ± 0.07</td>
<td>0.095 ± 0.007</td>
<td>1.66 ± 0.93</td>
<td>0.55 ± 0.03</td>
<td>0.032 ± 0.002</td>
<td>1.84 ± 0.21</td>
</tr>
<tr>
<td>20000316*</td>
<td>30048–55857</td>
<td>0.41 ± 0.04</td>
<td>0.027 ± 0.002</td>
<td>1.66 ± 0.30</td>
<td>0.45 ± 0.03</td>
<td>0.075 ± 0.005</td>
<td>1.48 ± 0.91</td>
<td>0.51 ± 0.02</td>
<td>0.034 ± 0.001</td>
<td>1.87 ± 0.22</td>
</tr>
<tr>
<td>20000318*</td>
<td>53934–69888</td>
<td>0.35 ± 0.06</td>
<td>—</td>
<td>—</td>
<td>0.41 ± 0.04</td>
<td>0.070 ± 0.005</td>
<td>1.60 ± 1.06</td>
<td>0.40 ± 0.04</td>
<td>0.041 ± 0.001</td>
<td>1.86 ± 0.22</td>
</tr>
</tbody>
</table>

Mean(across-jet) | 0.51 ± 0.15 | 0.033 ± 0.011 | 1.63 ± 0.80 | 0.49 ± 0.18 | 0.090 ± 0.031 | 1.81 ± 1.26 | 0.40 ± 0.18 | 0.029 ± 0.014 | 1.63 ± 0.47 |

Mean(along-jet)  | 0.41 ± 0.10 | 0.039 ± 0.033 | 1.91 ± 1.26 | 0.44 ± 0.18 | 0.086 ± 0.029 | 1.55 ± 0.81 | 0.41 ± 0.17 | 0.044 ± 0.037 | 1.93 ± 0.53 |

*indicates an along-jet flight; all others are across-jet.
SOLVE 20000311
Wind Vectors
1 Hz ER-2 Data
Across Polar Night Jet

Each plot represents 32 intervals.
SOLVE 20000311
Wind Vector
Differences

1 Hz ER-2 Data
Across Polar Night Jet

Each plot represents 32 intervals.
19870922 Wind Vector Differences
Centred At 61 S, 68 W
19870922 enddip - begindescent

Temperature: \( H_1 = 0.531 \pm 0.020, \ C_1 = 0.054 \pm 0.000, \ \alpha = 2.15 \pm 0.41 \)

Ozone: \( H_1 = 0.583 \pm 0.021, \ C_1 = 0.045 \pm 0.004, \ \alpha = 1.98 \pm 0.27 \)
* Observed $H(N_2O) = 5/9$ implies that vortex chemistry is operating in a turbulent space of 23/9 dimensions (not 2 or 3!)

* If the rate determining step is

$$-d[O_3]/dt = 2k[ClO]^2[M]$$

in conventional 3D (laboratory) space, what should it be in 23/9 dimensional space?

20000123: $H(ClO) = 0.76$
20000226: $H(ClO) = 0.46$
20000312: $H(ClO) = 0.32$

$M$, total pressure, scales like a passive scalar (tracer), so $H(M) = 0.56$

If a given molecular population is restricted to a space of reduced dimensionality, reaction rates should accelerate. So should the $[ClO]^2[M]$ be replaced by

$[ClO]^{2.2}[M]^{1.18}$ - since $2x(3 ÷ 2.76) = 2.20$ on 20000123
$[ClO]^{2.4}[M]^{1.18}$ on 20000226
$[ClO]^{2.55}[M]^{1.18}$ on 20000312?
Summary, Lecture 4.1

* Models do show scaling, but the exponents are not generally in agreement with observed ones.


* Contours need to be viewed as the approximate visual aids that they are, and not as a reliable basis for theory. Dissipation matters!

* Application of the law of mass action to vortex-wide averages operating inside a “containment vessel” is a flawed concept.