4.1

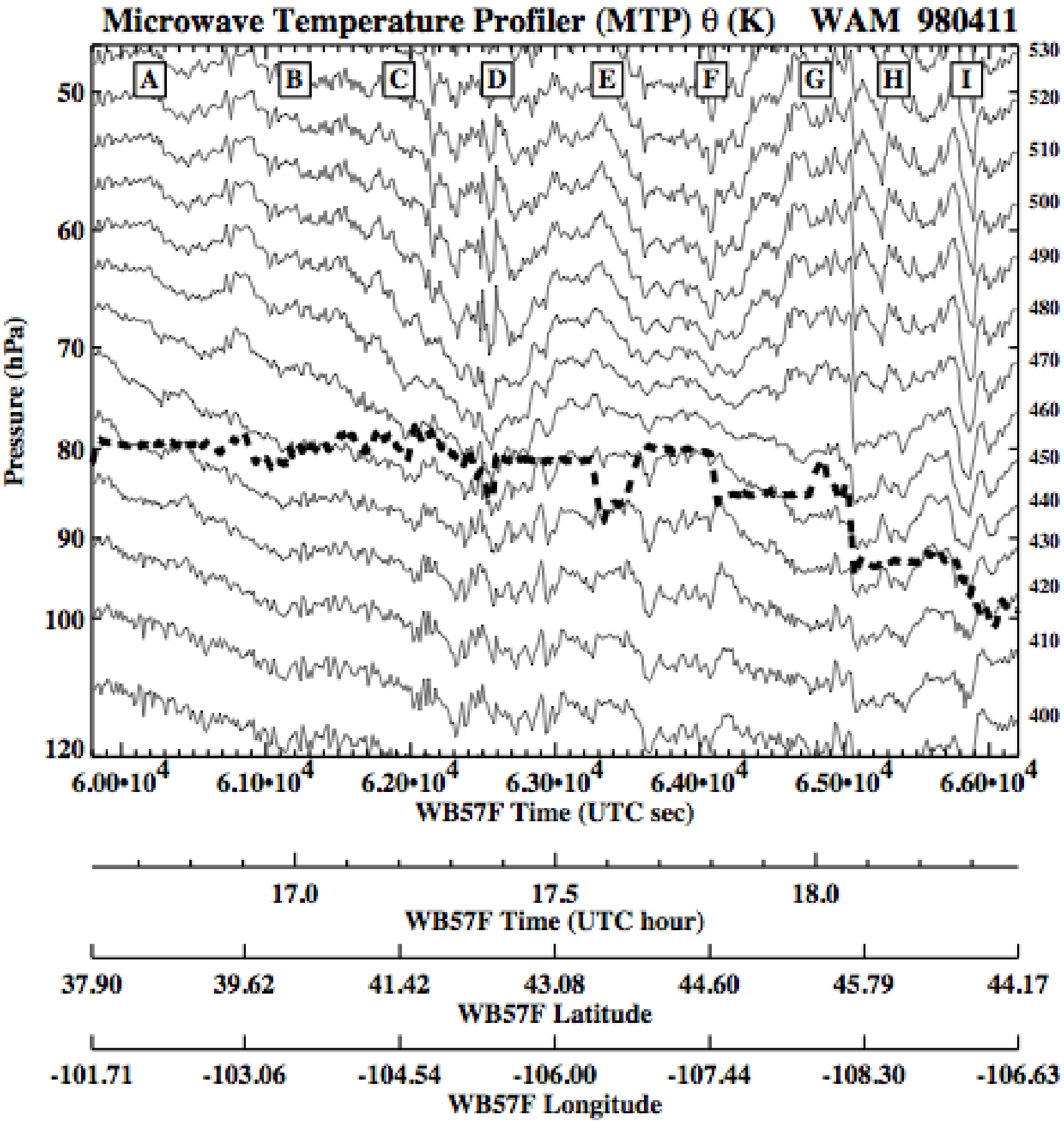
Lecture 4.1, Consequences [1] of observed scale invariance

* No isotropic turbulence or diffusion, no normal distributions, no stable layers, means converge but variances do not, no local thermodynamic equilibrium, **molecular properties must be accounted for by means other than the gas constant.**

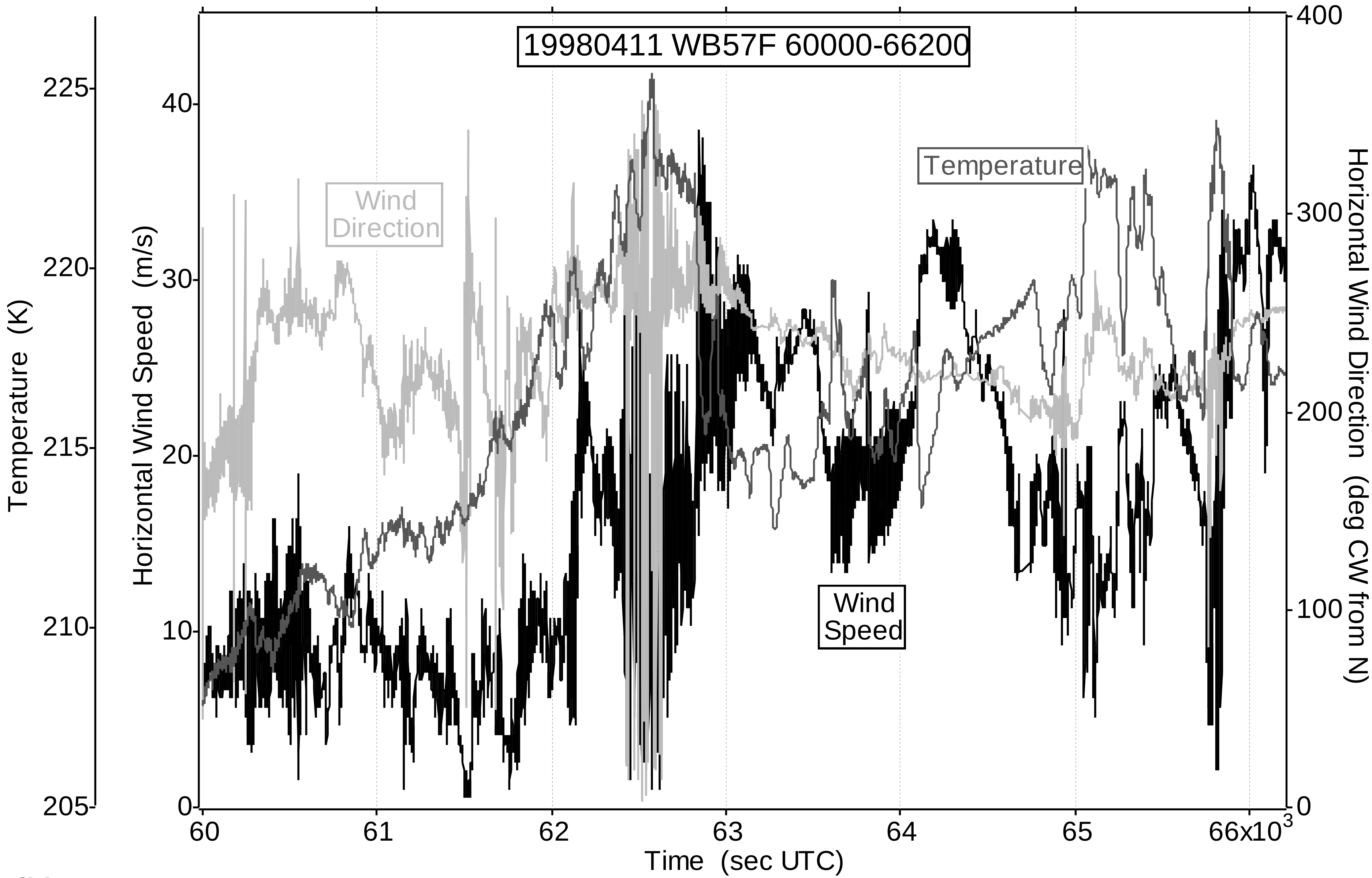
* These points imply difficulties in the formulation of numerical models. An example is given.

* Question: stratospheric polar vortex - containment vessel or flow reactor?

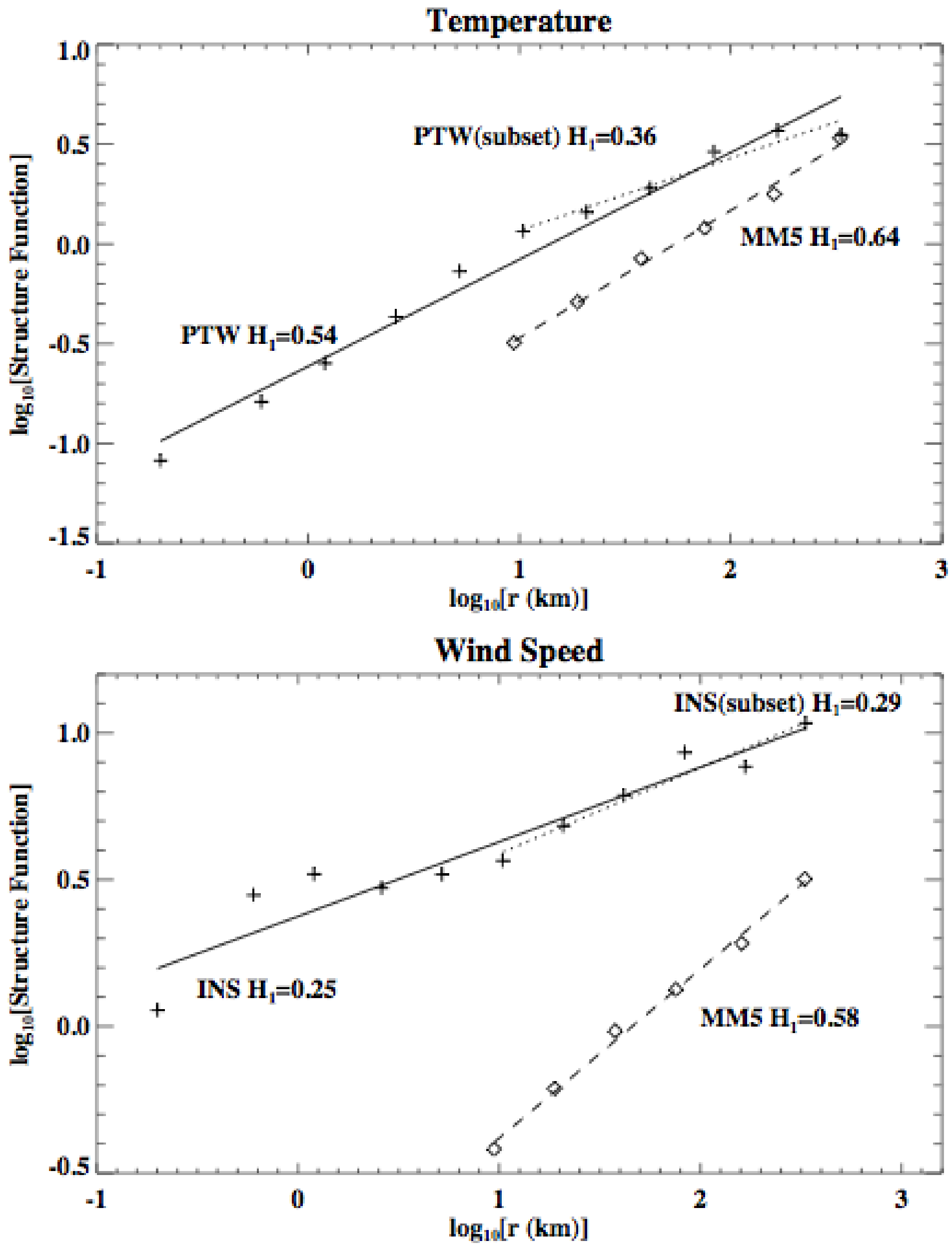
WB57F, Rocky Mountains 19980411. Severe turbulence & lee waves. Isentropes observed by MTP.



Lee waves near Riverton, Wyoming. Severe turbulence (WAM).



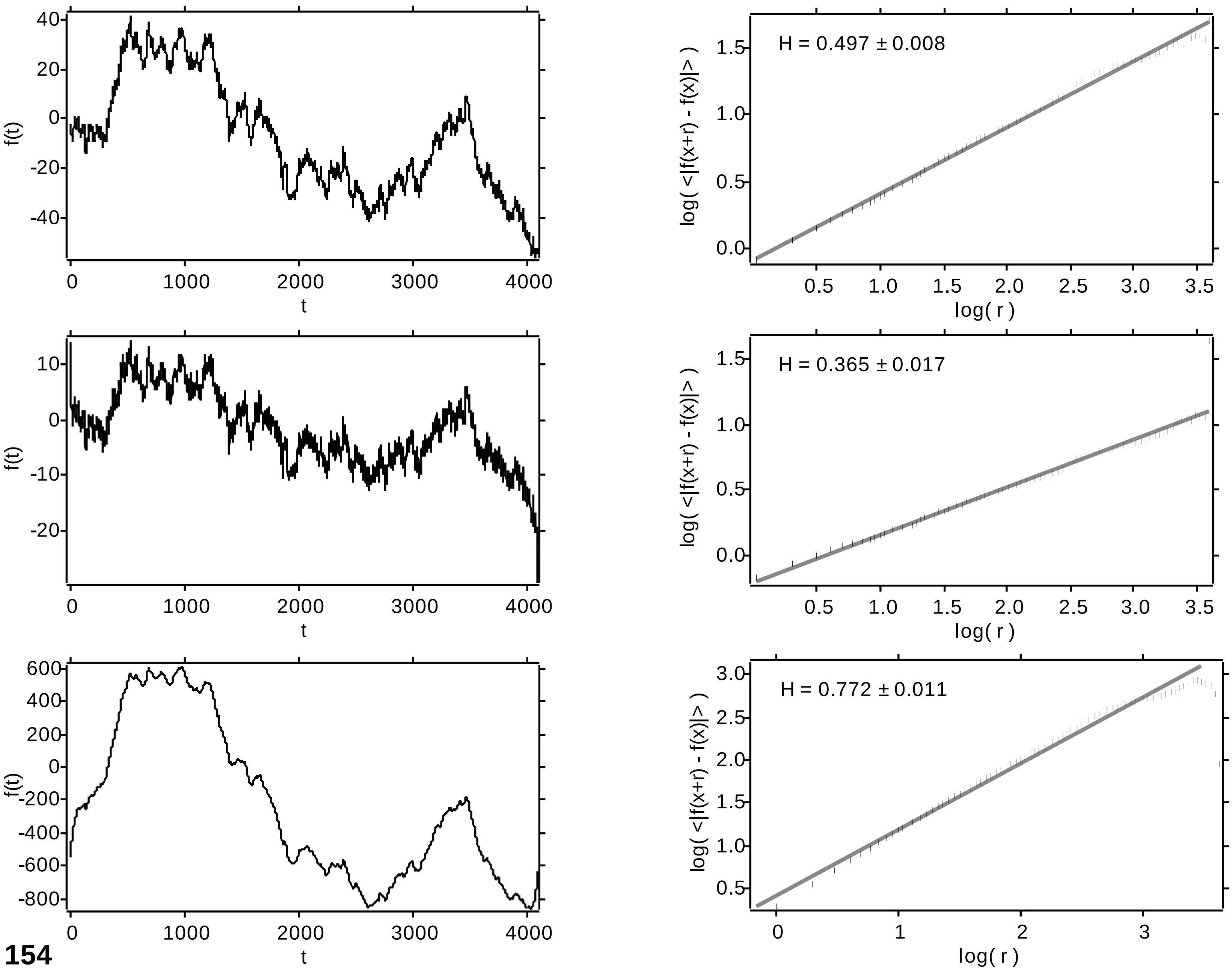
Scaling of WB57F observations and of MM5 simulation, 19980411.



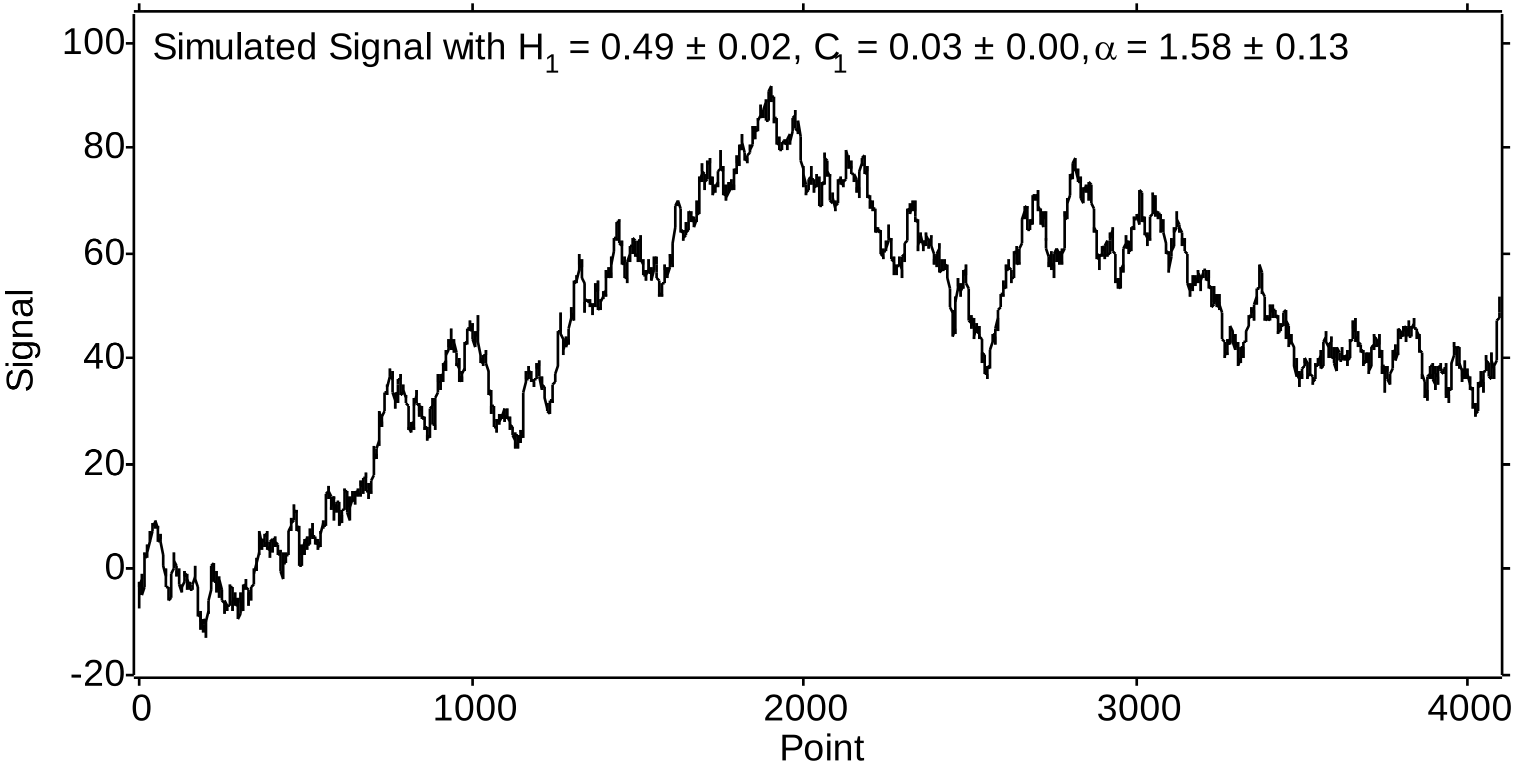
Scaling of WB57F and MM5, WAM Rocky Mountain Lee Wave Flight 19980411

Data set	H_1	H_2	H_6	β	$2H_2+1$	C_1
PTW Temperature	0.36	0.33	0.23	1.59	1.66	0.05
MM5 Temperature	0.64	0.58	0.50	2.00	2.16	0.09
INS Wind Speed	0.29	0.25	0.15	1.83	1.5	0.05
MM5 Wind Speed	0.58	0.53	0.45	2.05	2.06	0.09

Simulated monofractal signals: random, antipersistent & persistent.

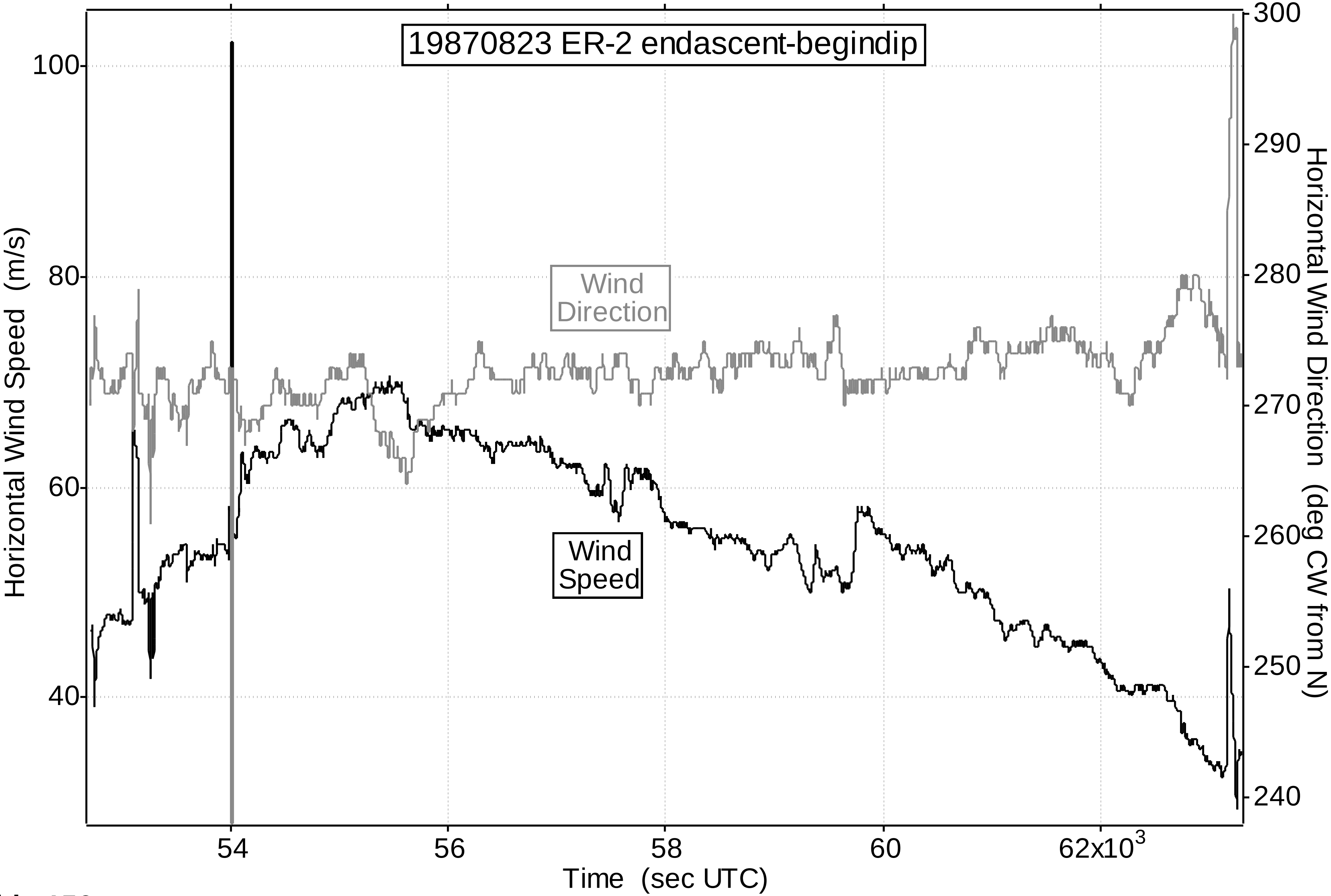


Simulated statistical multifractal signal, with typical observed values of generalized scale invariance exponents: conservation, intermittency and Lévy.

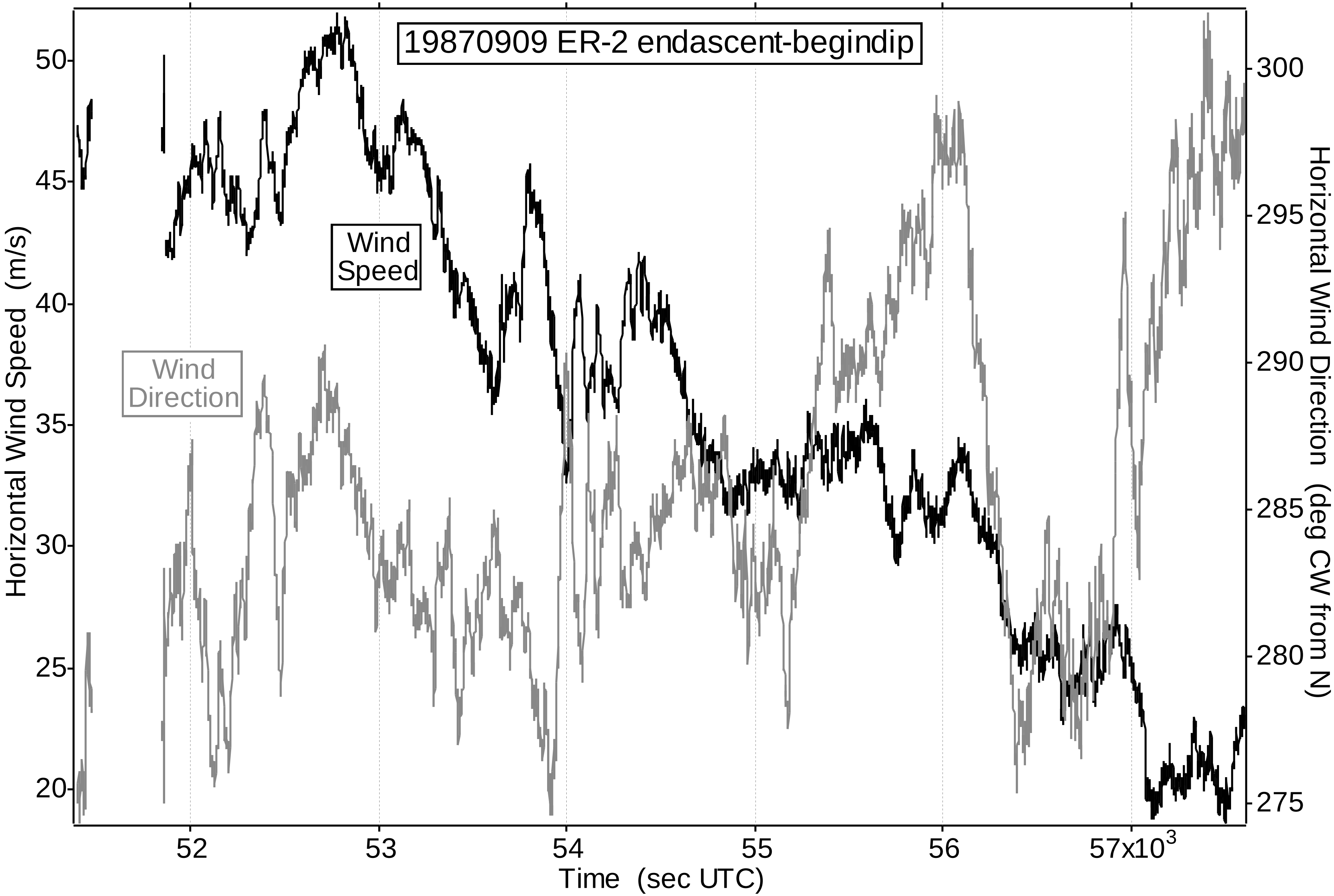


Although using typical exponents, the trace still does not "look like" actual observed ones.

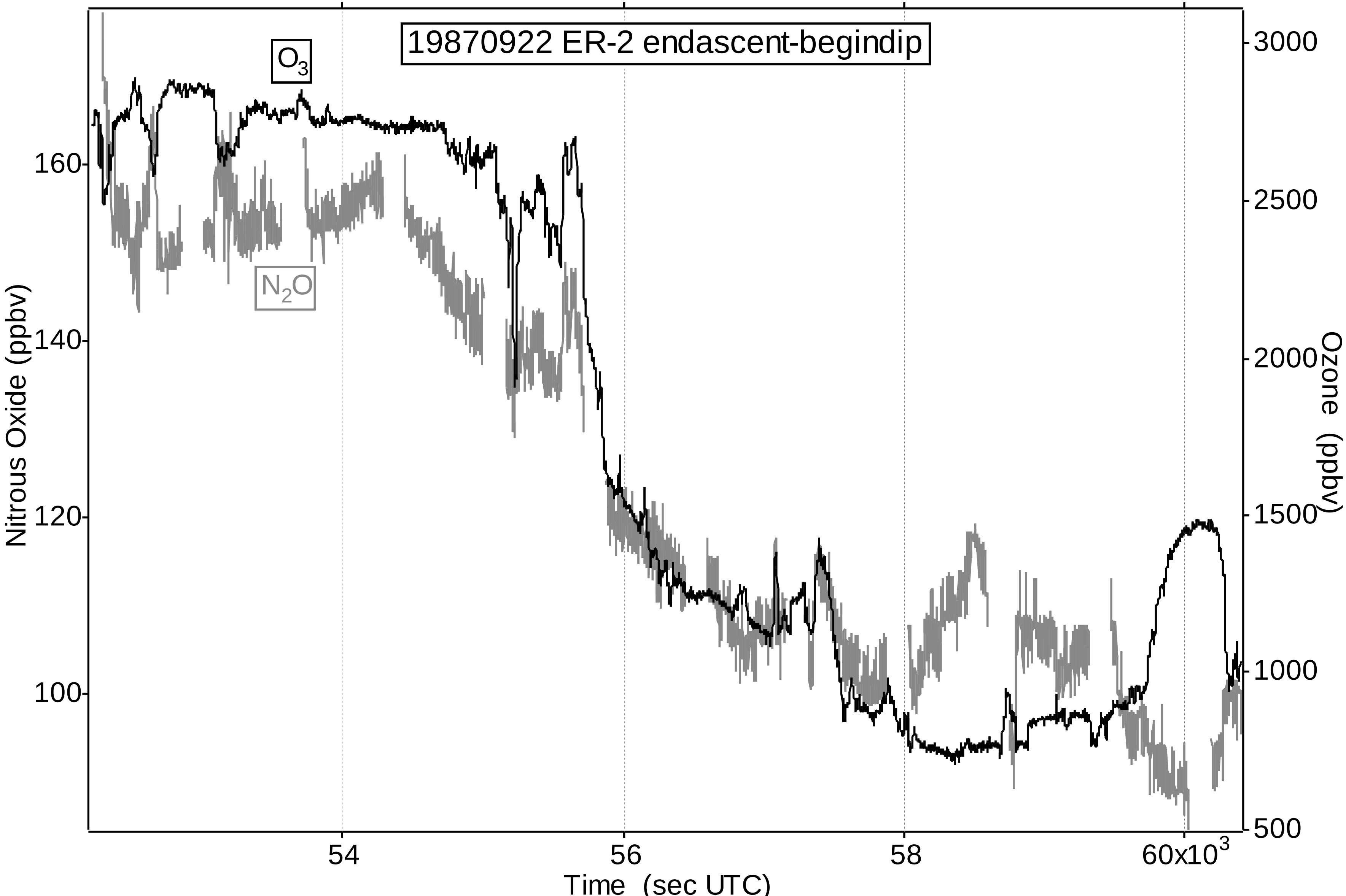
Intermittency, sharp gradients



Intermittency, sharp gradients



Intermittency, sharp gradients

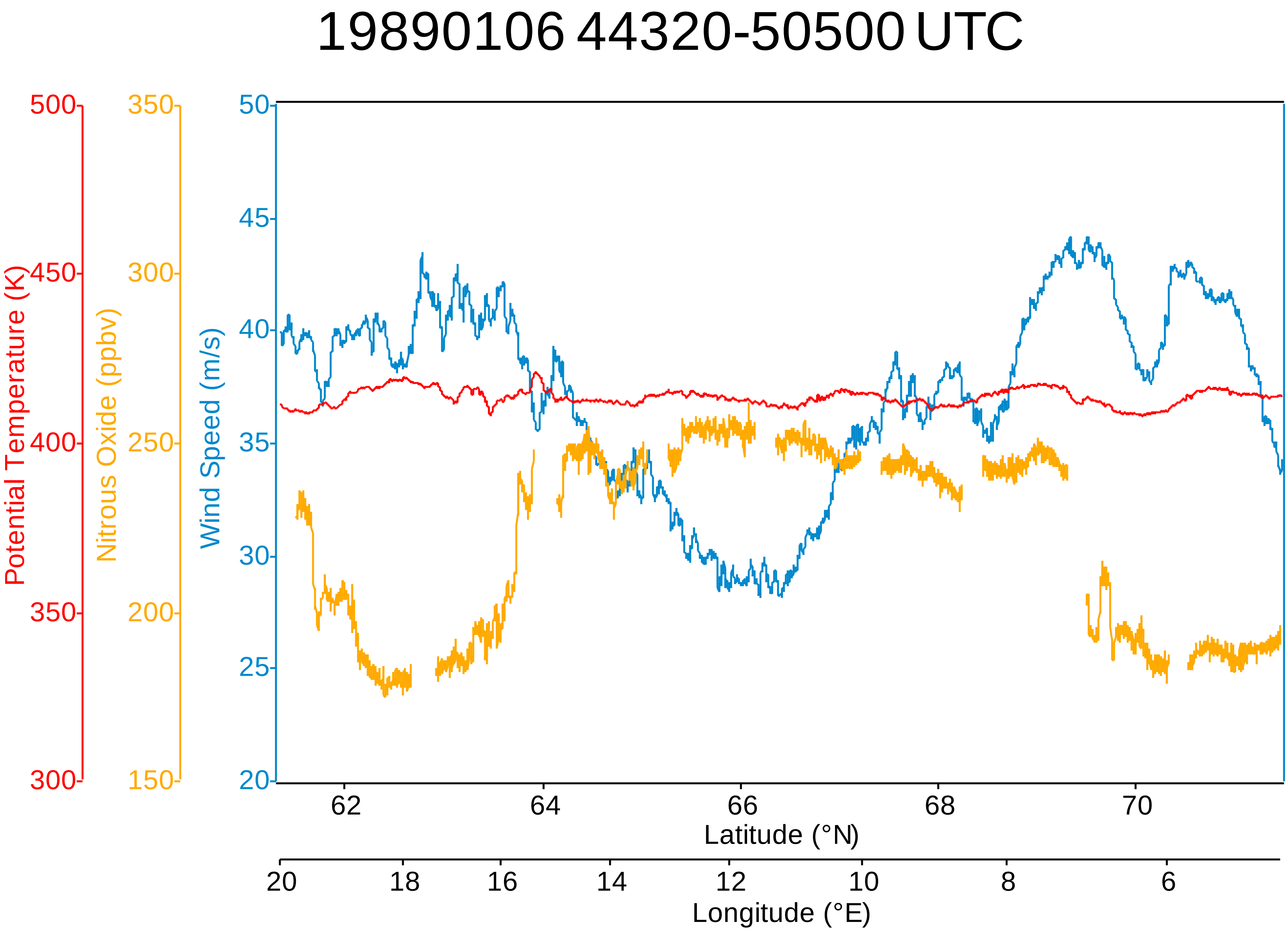


We now consider implications for a particular question: the degree of exchange across the edge of the stratospheric winter polar vortex, and the rates of reaction inside it.

* The scale invariance of the winds

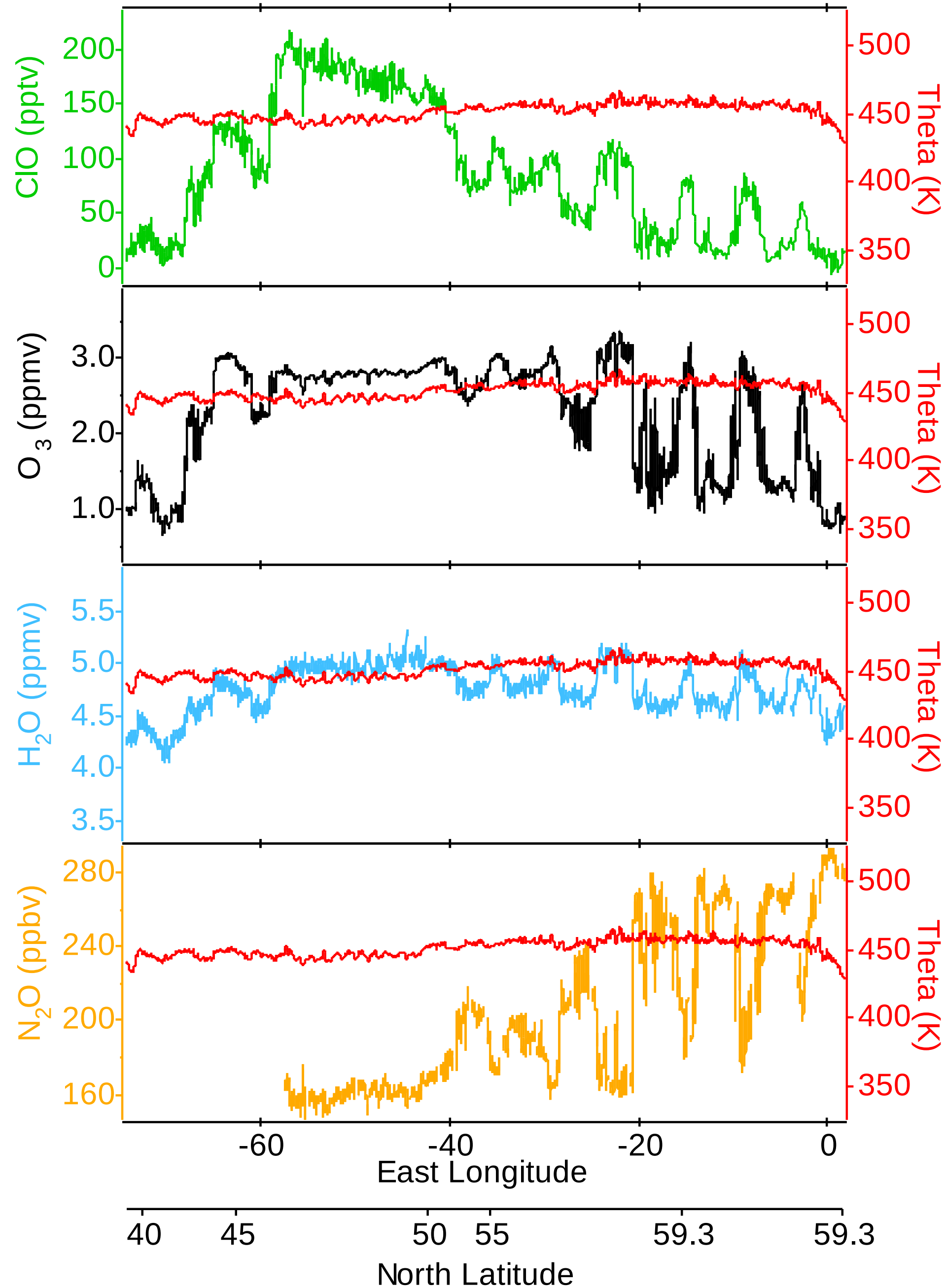
* The scale invariance of some of the chemical species

* Application of the law of mass action

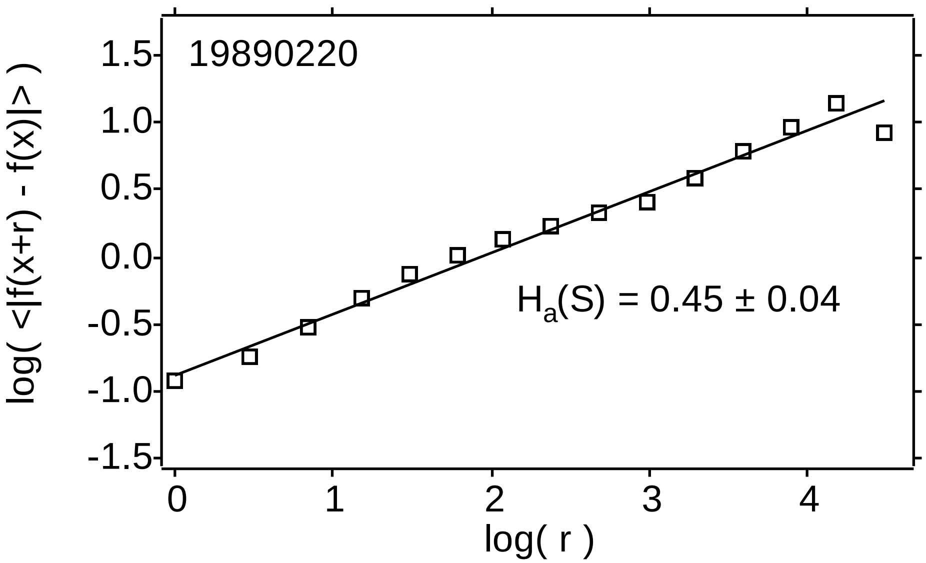

19890106 44320-50500 UTC
Potential Temperature (K)
Nitrous Oxide (ppbv)
Wind Speed (m/s)
300
350
400
450
500
150
200
250
300
350
20
25
30
35
40
45
50
62
64
66
68
70
Latitude (°N)
20
18
16
14
12
10
8
6
Longitude (°E)

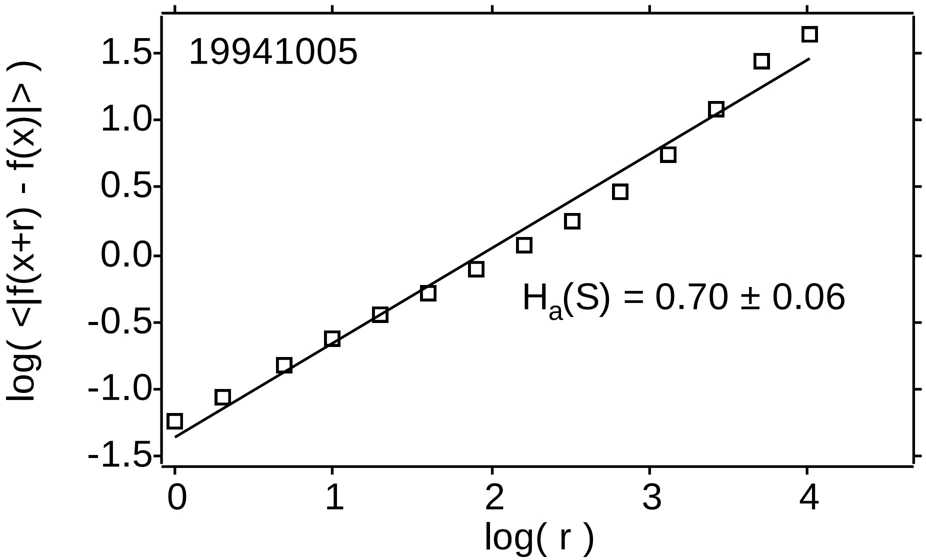
Slide 160

19890220 endascent - begindescent



Scaling Exponents for Wind Speed





Scaling exponents H_1, C_1 and α for SOLVE ER-2, full precision at 5 Hz

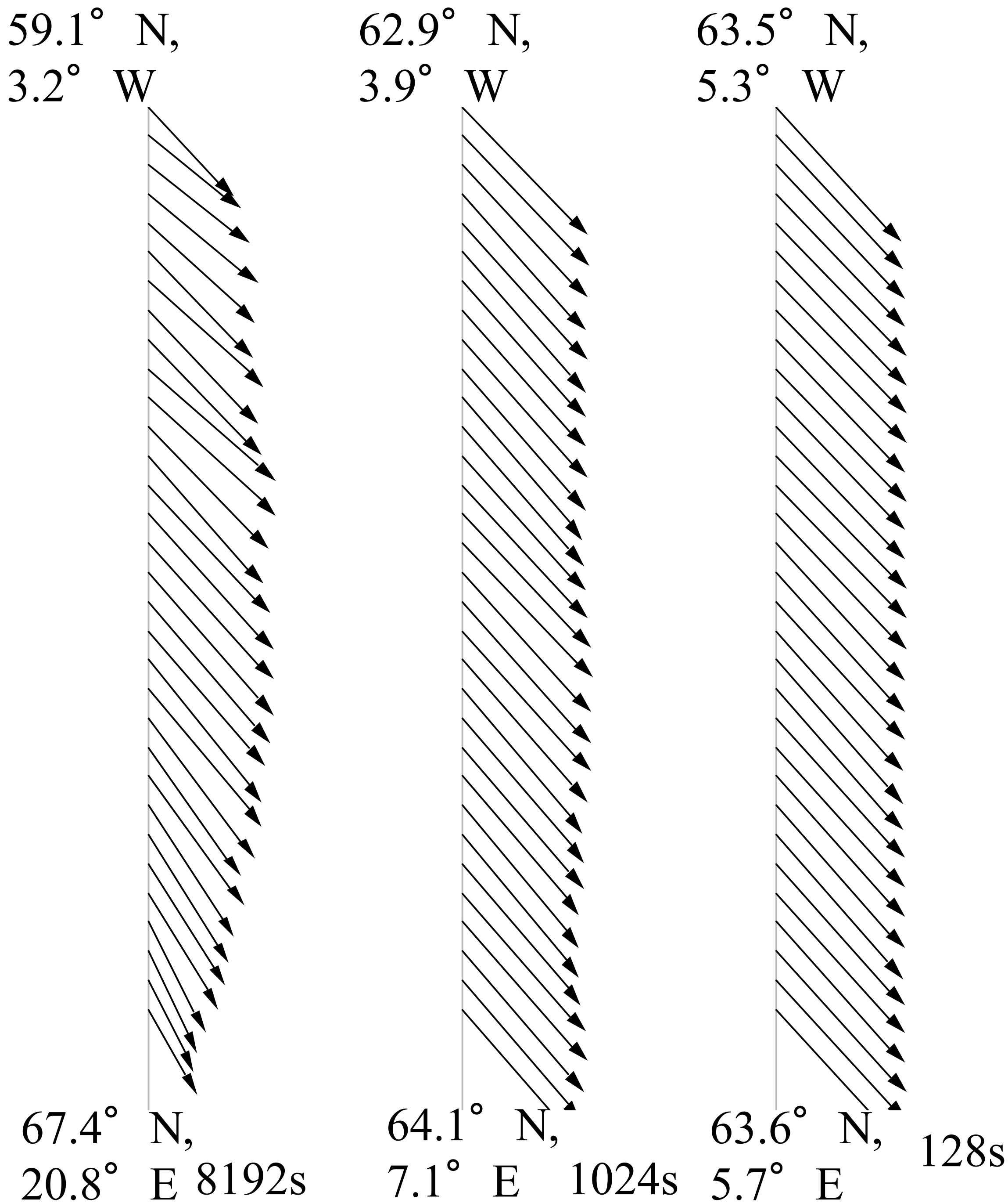
Date (yyymmdd)	Start & stop times	$H_a(S)$±c.i.	$C_1(S)$±c.i.	$\alpha(S)$±c.i.	$H_a(T)$±c.i.	$C_1(T)$±c.i.	$\alpha(T)$±c.i.	$H_a(O_3)$±c.i.	$C_1(O_3)$±c.i.	$\alpha(O_3)$±c.i.
20000111*	50118-54288	0.45 ± 0.05	0.042 ± 0.002	1.45 ± 0.33	0.40 ± 0.10	0.088 ± 0.006	1.34 ± 0.94	0.39 ± 0.06	0.070 ± 0.002	1.80 ± 0.15
20000111*	54288-59538	0.43 ± 0.05	0.060 ± 0.001	2.31 ± 0.74	0.39 ± 0.08	0.102 ± 0.004	1.48 ± 0.84	0.44 ± 0.05	0.052 ± 0.002	1.89 ± 0.15
20000114	46800-63931	0.52 ± 0.04	0.034 ± 0.001	1.72 ± 0.14	0.59 ± 0.05	0.067 ± 0.004	1.48± 0.56	0.44 ± 0.06	—	—
20000120	37553-47828	0.47 ± 0.06	0.026 ± 0.002	1.53 ± 0.21	0.48 ± 0.06	0.094 ± 0.007	1.78 ± 0.96	0.31 ± 0.04	0.023 ± 0.002	1.56 ± 0.15
20000123*	31017-38648	0.41 ± 0.05	0.027 ± 0.002	2.22 ± 1.39	0.57 ± 0.05	0.096 ± 0.007	1.84 ± 1.02	0.31 ± 0.06	0.023 ± 0.002	2.21 ± 0.92
20000127	45647-52267	0.43 ± 0.07	0.044 ± 0.002	2.38 ± 1.30	0.44 ± 0.07	0.085 ± 0.005	1.91 ± 1.97	0.41 ± 0.05	—	—
20000131	38199-42764	0.54 ± 0.05	0.037 ± 0.002	1.39 ± 0.24	0.54 ± 0.04	0.114 ± 0.008	1.83 ±1.77	0.37 ± 0.07	0.027 ± 0.002	1.54 ± 0.15
20000202	42000-53500	0.54 ± 0.04	0.026 ± 0.002	1.37 ± 0.16	0.51 ± 0.05	0.094 ± 0.007	1.72 ± 1.85	0.41 ± 0.06	0.023 ± 0.002	1.39 ± 0.13
20000203	69353-72748	0.39 ± 0.04	0.038 ± 0.003	1.46 ± 0.22	0.31 ± 0.10	0.064 ± 0.005	1.11 ± 0.33	0.38 ± 0.08	0.037 ± 0.002	1.54 ± 0.25
20000226	31000-43000	0.57 ± 0.05	0.034 ± 0.002	1.38 ± 0.15	0.49 ± 0.05	0.076 ± 0.006	2.75 ± 2.00	0.39 ± 0.02	0.024 ± 0.002	1.53 ± 0.19
20000305	41747-52392	0.52 ± 0.03	0.032 ± 0.002	2.12± 1.23	0.55 ± 0.05	0.090 ± 0.006	2.30 ± 1.82	0.35 ± 0.06	0.026 ± 0.002	1.78 ± 0.21
20000307	37000-43000	0.46 ± 0.04	0.033 ± 0.002	1.43 ± 0.20	0.51 ± 0.05	0.104 ± 0.008	1.60 ± 1.00	0.26 ± 0.02	0.025 ± 0.002	1.33 ± 0.13
20000311	31524-39509	0.56 ± 0.06	0.035 ± 0.002	1.71 ± 0.63	0.60 ± 0.04	0.101 ± 0.008	1.59 ± 1.91	0.55 ± 0.04	0.042 ± 0.002	1.94 ± 0.23
20000311	42354-52389	0.62 ± 0.06	0.031 ± 0.002	1.55 ± 0.20	0.46 ± 0.05	0.093 ± 0.006	1.92 ± 1.71	0.55 ± 0.02	0.034 ± 0.002	1.82 ± 0.18
20000312	51934-58549	0.56 ± 0.04	0.030 ± 0.002	1.56 ± 0.18	0.44 ± 0.07	0.095 ± 0.007	1.66 ± 0.93	0.35 ± 0.03	0.032 ± 0.002	1.84 ± 0.21
20000316*	30048-55857	0.41 ± 0.04	0.027 ± 0.002	1.66 ± 0.30	0.45 ± 0.03	0.075 ± 0.005	1.48 ± 0.91	0.51 ± 0.02	0.034 ± 0.001	1.87 ±0.22
20000318*	53934-69882	0.35 ± 0.06	—	—	0.41 ± 0.04	0.070 ± 0.005	1.60 ± 1.06	0.40 ± 0.04	0.041 ± 0.001	1.86 ± 0.22
Mean(across-jet)		0.51 ± 0.15	0.033 ± 0.011	1.63 ± 0.80	0.49 ± 0.18	0.090 ± 0.031	1.81 ± 1.26	0.40 ± 0.18	0.029 ± 0.014	1.63 ± 0.47
Mean(along-jet)		0.41 ± 0.10	0.039 ± 0.033	1.91 ± 1.26	0.44 ± 0.18	0.086 ± 0.029	1.55 ± 0.81	0.41 ± 0.17	0.044 ± 0.037	1.93 ± 0.53

*indicates an along-jet flight; all others are across-jet.

SOLVE 20000311
Wind Vectors

1 Hz ER-2 Data
Across Polar Night
Jet

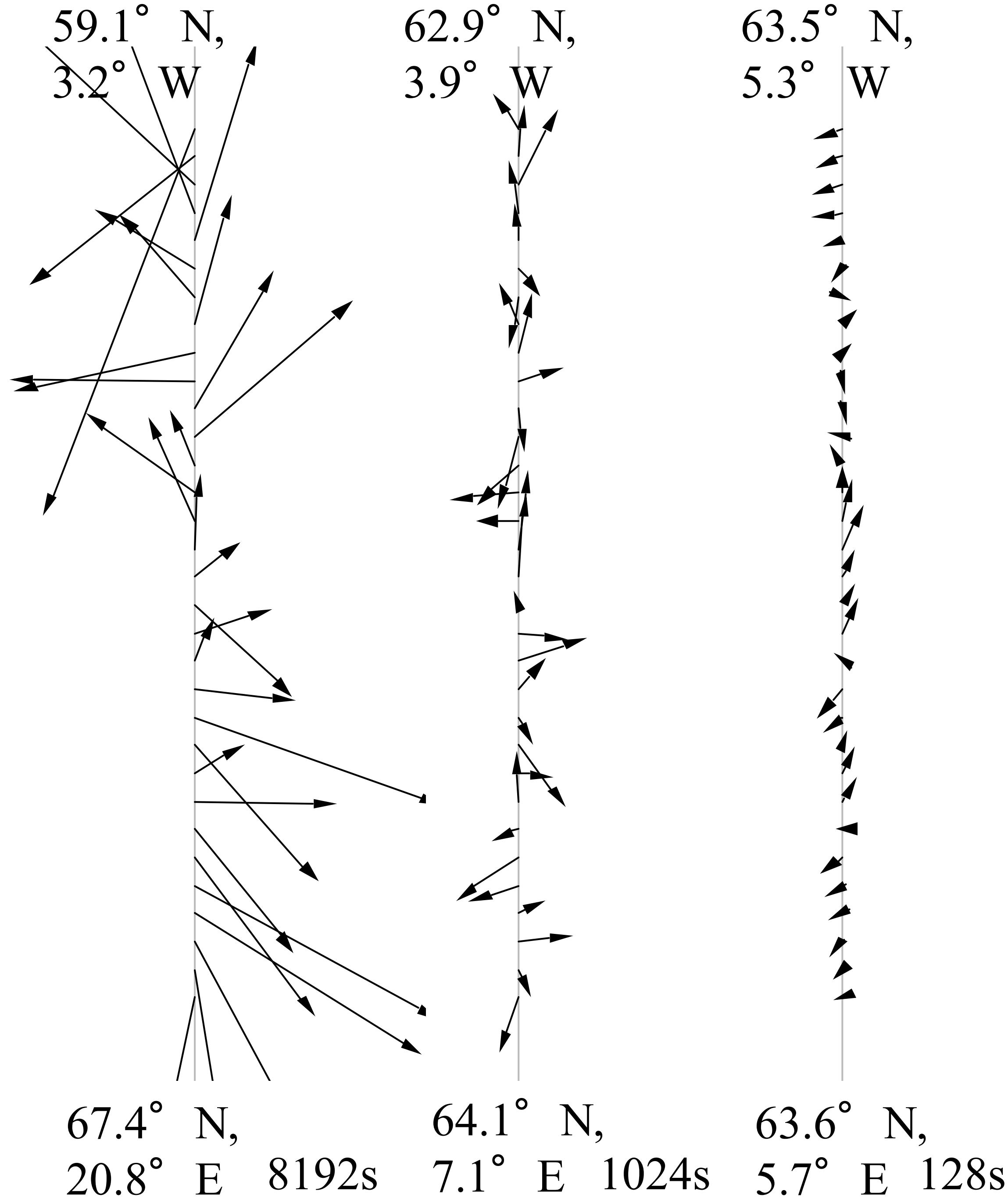
Each plot represents 32 intervals.



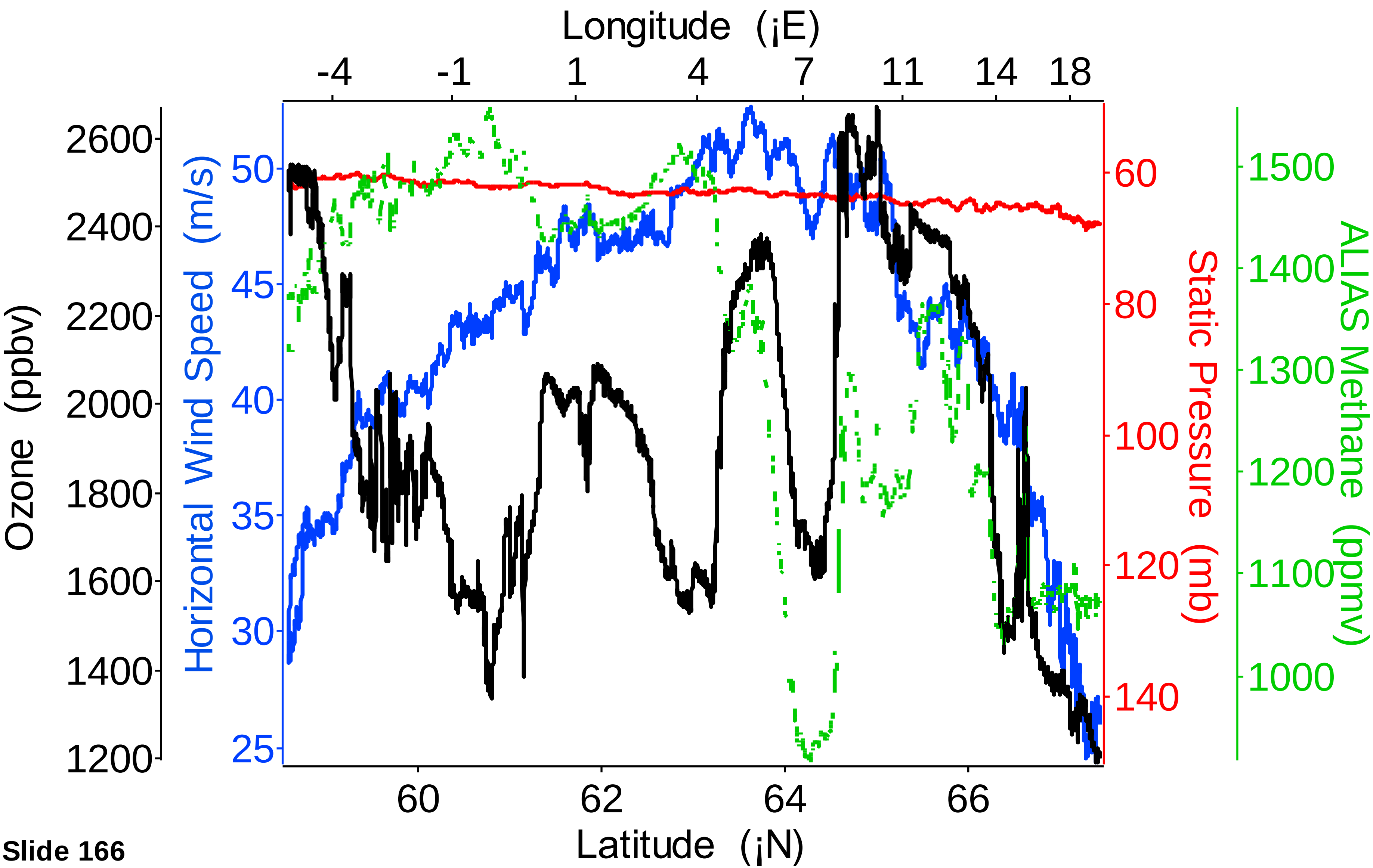
SOLVE 20000311
Wind Vector
Differences

1 Hz ER-2 Data
Across Polar Night
Jet

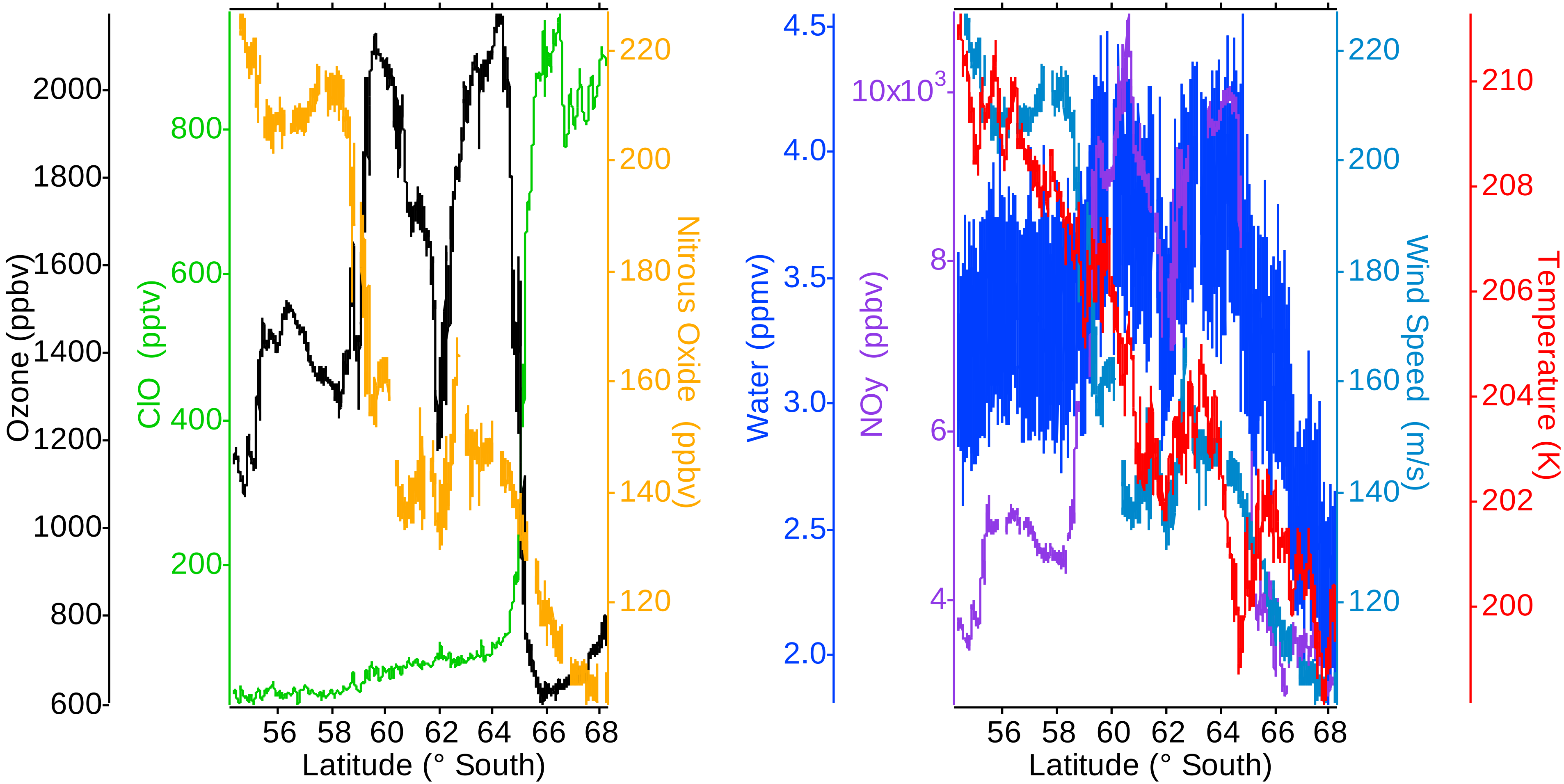
Each plot
represents 32
intervals.



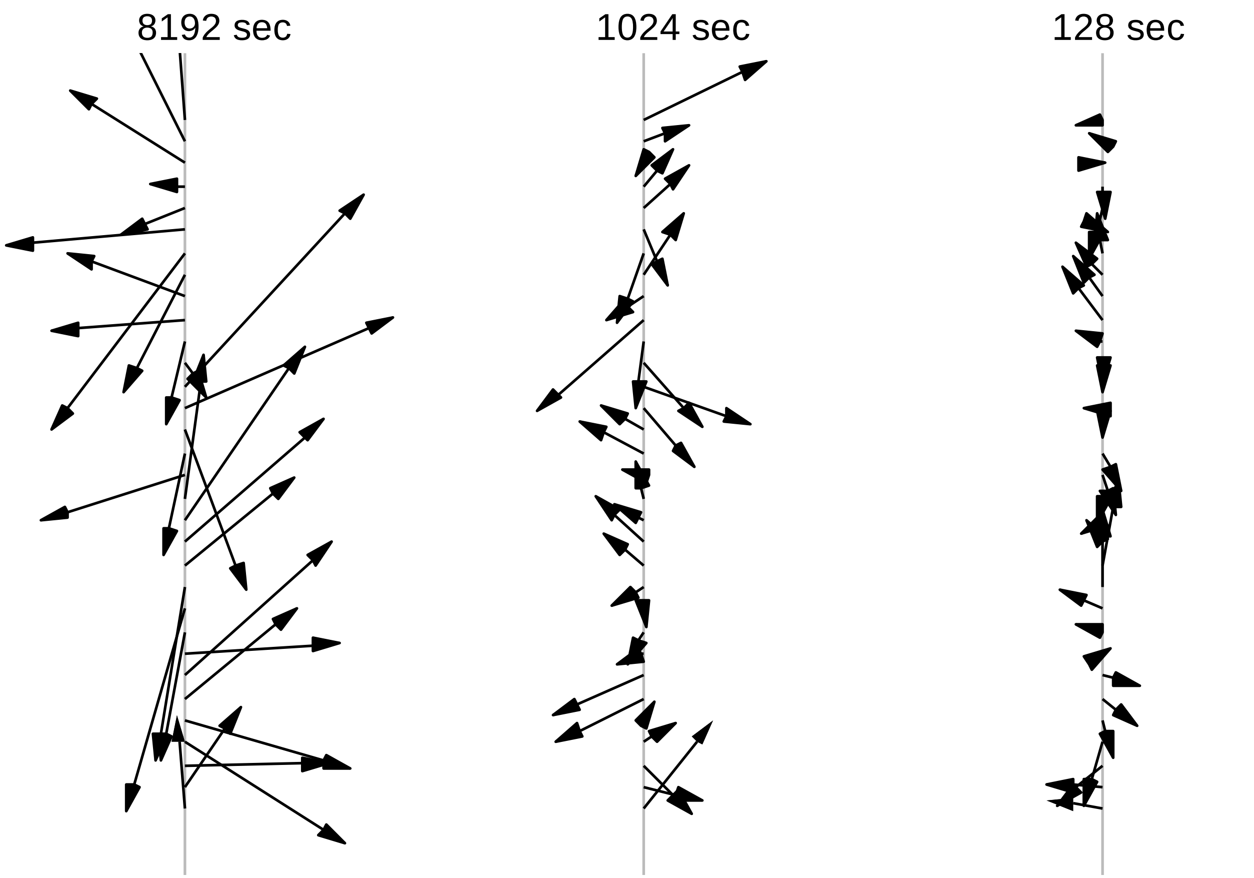
SOLVE ER-2 20000311
endascent - begindip



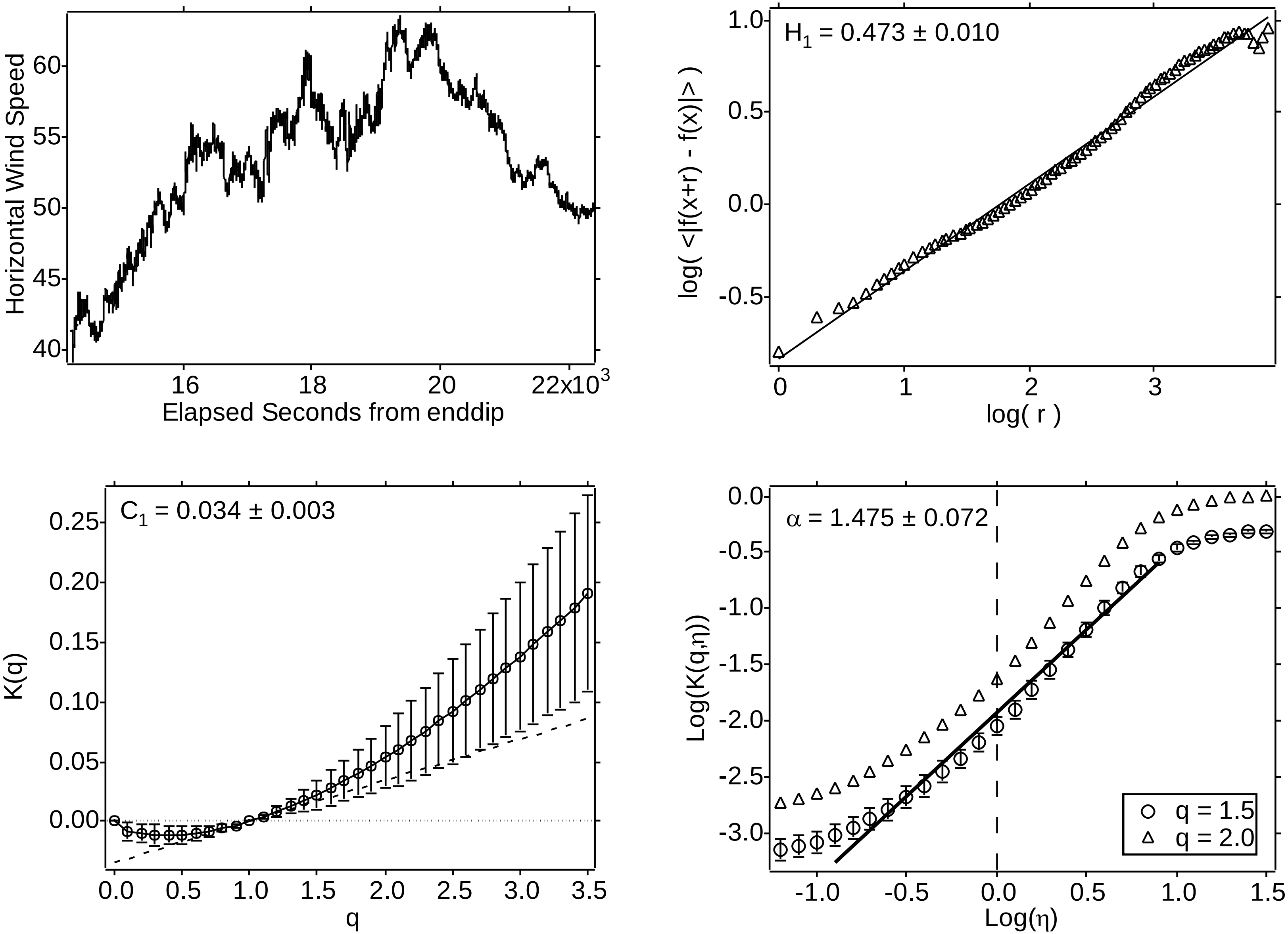
19870922 enddip - begindescent



19870922 Wind Vector Differences
Centred At 61 S, 68 W



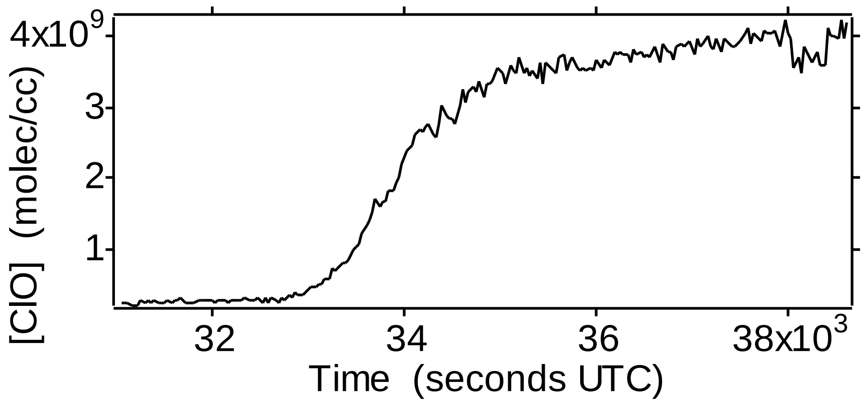
19870922 enddip - begindescent

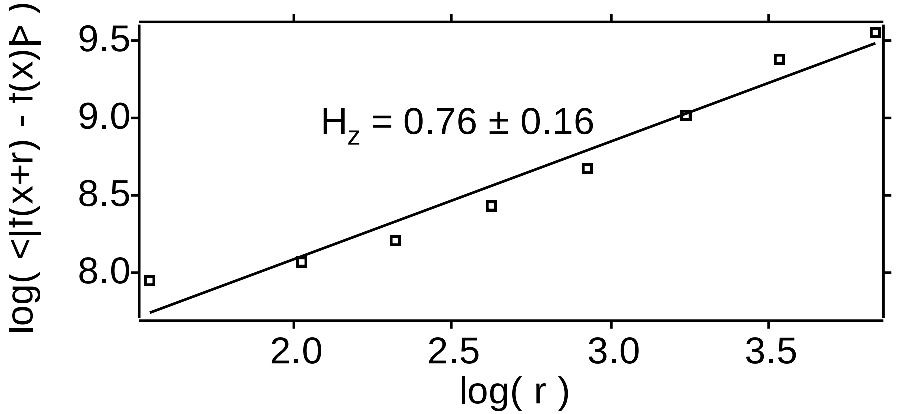


Temperature: H_1=0.531 ± 0.020, C_1=0.054 ± 0.000, α=2.15 ± 0.41

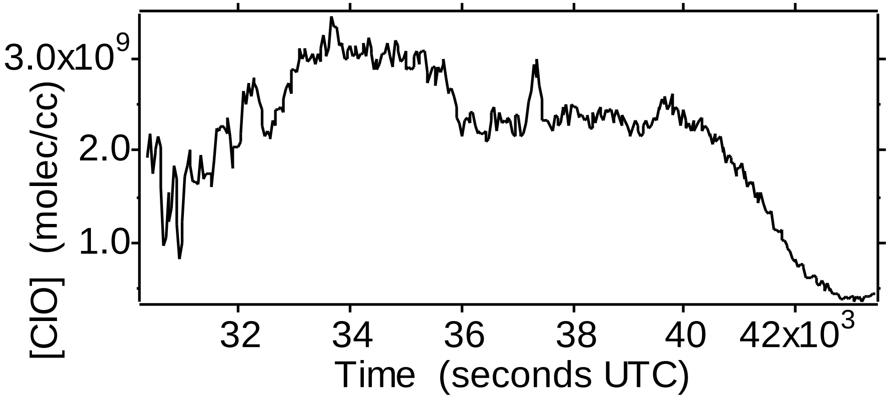
Ozone: H_1=0.583 ± 0.021, C_1=0.045 ± 0.004, α=1.98 ± 0.27

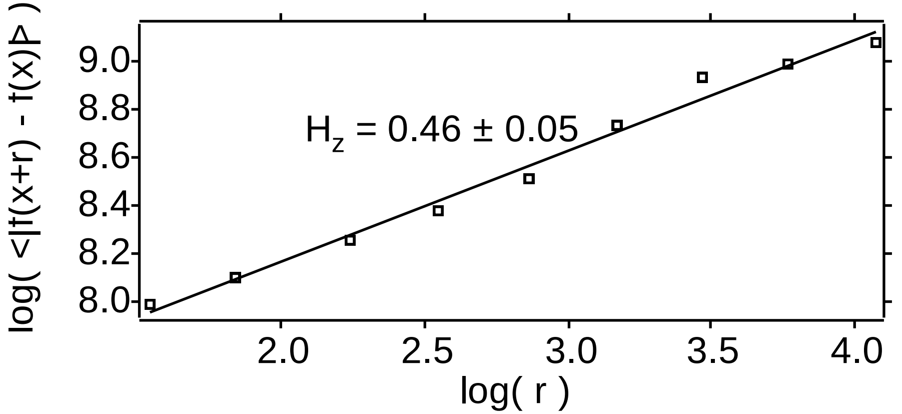
20000123



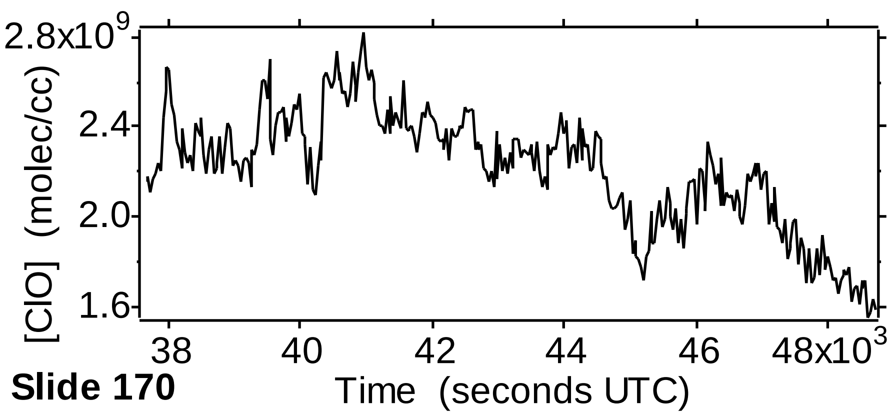


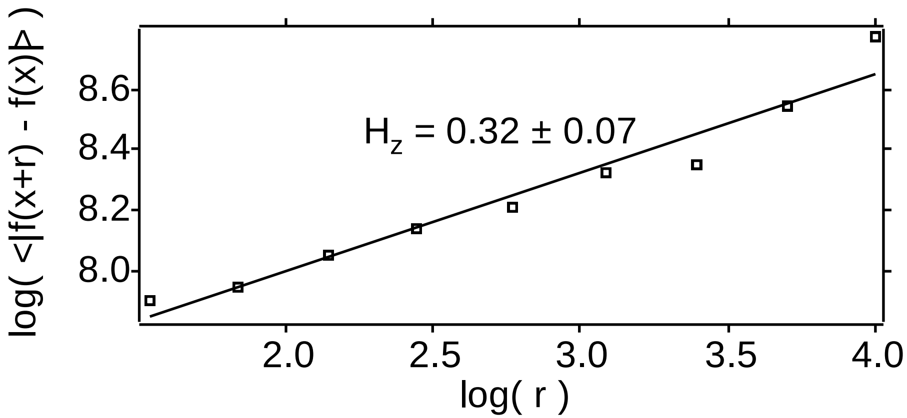
20000226





20000312





* Observed $H(N_2O) = 5/9$ implies that vortex chemistry is operating in a turbulent space of 23/9 dimensions (**not** 2 or 3!)

* If the rate determining step is

$$-d[O_3]/dt = 2k[ClO]^2[M]$$

in conventional 3D (laboratory) space, what should it be in 23/9 dimensional space?

20000123: $H(ClO) = 0.76$
20000226: $H(ClO) = 0.46$
20000312: $H(ClO) = 0.32$

M, total pressure, scales like a passive scalar (tracer), so
$H(M) = 0.56$

If a given molecular population is restricted to a space of reduced dimensionality, reaction rates should accelerate.
So should the $[ClO]^2[M]$ be replaced by
$[ClO]^{2.2}[M]^{1.18}$ - since $2x(3 \div 2.76) = 2.20$ on 20000123
$[ClO]^{2.4}[M]^{1.18}$ on 20000226
$[ClO]^{2.55}[M]^{1.18}$ on 20000312?

Summary, Lecture 4.1

* Models do show scaling, but the exponents are not generally in agreement with observed ones.

See also Lovejoy & Schertzer, Space-time cascades and the scaling of ECMWF analyses: fluxes and fields. *J. Geophys. Res.,* **116,** D14117, doi: 10.1029/2011JD015654.

* Contours need to be viewed as the approximate visual aids that they are, and not as a reliable basis for theory. Dissipation matters!

* Application of the law of mass action to vortex-wide averages operating inside a “containment vessel” is a flawed concept.