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Reflection of light from particulate media with irregularly shaped particles

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Abstract

The paper is devoted to the derivation of a simple approximate equation for the reflection function of a semi-infinite nonabsorbing medium composed of irregularly shaped particles. It is assumed that particles in a scattering medium can be modelled as fractal objects (Koch fractals of the second generation). The refractive index of ice in visible was used for calculations. Therefore, results presented can be used to model light reflection from thick snow fields and crystalline clouds. The accuracy of the parameterization derived is studied using the numerical solution of the radiative transfer equation. It was found that the approximation can be used with the accuracy better than 3% for near nadir observation conditions.

Keywords: Radiative transfer; Snow; Light reflection

1. Introduction

Light scattering media composed of irregularly shaped particles are of a common occurrence. The examples are snow fields, ice and mixed clouds, soils, grass, whitecaps, and desert sand, to name a few. The application of standard tools of the reflectance spectroscopy [1] to such media encounters a number of difficulties. Most important are the peculiar shape of particles, which scatter light, and, as a rule, their large concentrations in a unit volume of a scattering medium.

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The aim of this study is to propose a simple parameterization, which can be used to describe main angular reflectance features of such objects. This goal requires a number of simplifying assumptions. First of all, we assume that there is no light absorption in the medium. Also, we consider only semi-infinite media. Possible close-packed effects are neglected. Therefore, the reflection function is determined completely by the phase function of the medium. In this study, we will use the phase function of the fractal ice particles given in [2] to find the coefficients in the general equation derived.

So our results are directly relevant to modelling of snow optical properties. This issue is of a considerable importance due to new findings on climatic effects of snow [3]. However, we believe that the results obtained can also be used for the modelling of light reflectance from other media with irregularly shaped particles.

The procedure of derivations is quite straightforward. We separate the contribution of single light scattering from the rest. Then we use the exact analytical result for the single light scattering contribution and parameterize the multiple light scattering contribution. The influence of the azimuthal angle on the angular distribution of photons scattered more than once is ignored.

The results of the work can be used for the ground reflection removal in aerosol retrievals using data from the multi-angle satellite radiometers [4]. Also equations derived can be used as convenient kernels in the algorithms to retrieve surface properties from space [5].

2. Theory

The reflection function of a semi-infinite layer $R(\zeta,\eta,\varphi)$ (ζ is the cosine of the incidence angle ϑ_1, η is the cosine of the observation angle ϑ_2, and φ is the relative azimuth of incident and reflected beams) can be written as [2]

$$R(\zeta,\eta,\varphi) = \bar{R}(\zeta,\eta) + R_{ss}(\zeta,\eta,\varphi) + 2\sum_{m=1}^{\infty} R_m(\zeta,\eta)\cos(m\varphi) \tag{1}$$

where

$$\bar{R}(\zeta,\eta) = \frac{1}{2\pi}\int_0^{2\pi} [R(\zeta,\eta,\varphi) - R_{ss}(\zeta,\eta,\varphi)]\,\mathrm{d}\varphi \tag{2}$$

and

$$R_m(\zeta,\eta) = \frac{1}{2\pi}\int_0^{2\pi} [R(\zeta,\eta,\varphi) - R_{ss}(\zeta,\eta,\varphi)]\cos(m\varphi)\,\mathrm{d}\varphi. \tag{3}$$

R_{ss} gives the reflection function for singly scattered light, $\bar{R}(\zeta,\eta)$ gives the reflection function averaged over the azimuth (after subtraction of the single scattering contribution) and $R_m(\zeta,\eta)$ is the mth Fourier coefficient of the reflection function (also after the subtraction of the single scattering contribution).

Eq. (1) is a convenient starting point for the parameterization purposes because it decouples (ζ,η)-dependences from the azimuthal dependence φ. It is much easier to parameterize functions $\bar{R}(\zeta,\eta) = \bar{R}(\eta,\zeta)$ and $R_m(\zeta,\eta) = R_m(\eta,\zeta)$ than the three-parameter function $R(\zeta,\eta,\varphi)$ [6].

The third term in Eq. (1) is of no importance for nadir observations, where the azimuth φ plays no role due to the symmetry of the problem. This contribution can be easily found analytically. It is given by the following equation [7]:

$$R_{ss}(\zeta,\eta,\varphi)=\frac{p(\theta)}{4(\zeta+\eta)}, \tag{4}$$

where $p(\theta)$ is the phase function and

$$\theta=\arccos(-\zeta\eta+\sqrt{(1-\zeta^2)(1-\eta^2)}\cos\varphi) \tag{5}$$

is the scattering angle.

Ignoring the third term in Eq. (1), we obtain

$$R(\zeta,\eta,\varphi)=\bar{R}(\zeta,\eta)+\frac{p(\theta)}{4(\zeta+\eta)}. \tag{6}$$

In essence, Eq. (6) states that only the azimuthal dependence of singly scattered light plays a primary role in the overall dependence.

The parameterization of the function $\bar{R}(\zeta,\eta)$ can be achieved as follows. First of all, we note that it follows for isotropic scattering ($p(\theta)\equiv 1$) [7]:

$$R(\zeta,\eta)=\frac{\Phi(\zeta)\Phi(\eta)}{4(\zeta+\eta)}. \tag{7}$$

The function $\Phi(\zeta)$ can be approximated by the following linear dependence [7]:

$$\Phi(\zeta)=\alpha(1+2\zeta), \tag{8}$$

where $\alpha=4\sqrt{3}/7$. Clearly, the azimuth does not enter equations for the isotropic scattering case.

The substitution of Eq. (8) into Eq. (7) gives

$$R(\zeta,\eta)=\frac{A+B(\zeta+\eta)+C\zeta\eta}{4(\zeta+\eta)}, \tag{9}$$

where $A=48/49\approx 1$, $B=2A\approx 2$, $C=4A\approx 4$.

Now, we make the assumption that Eq. (9) holds for $\bar{R}(\zeta,\eta)$ in Eq. (6) for arbitrary $P(\theta)$. However, coefficients A, B, and C then depend on the phase function of a particular medium. In the first approximation, only the dependence of A, B, and C on the asymmetry parameter

$$g=\frac{1}{2}\int_0^{\pi}p(\theta)\sin\theta\cos\theta\,\mathrm{d}\theta \tag{10}$$

may be considered as it was done, e.g., in [6] for a particular case of the Henyey–Greenstein phase function. Note that the parameter g for crystalline media, considered here, only weakly varies in visible ($g\approx 0.75$) and we can assume the same constants A, B, and C for ice crystalline media with arbitrary crystal habits.

Coefficients A, B, and C are found using the linear fitting of functions $\Pi(\zeta,\eta)=(\zeta+\eta)\bar{R}(\zeta,\eta)$ for a fixed η. The nonlinearity of these functions can be neglected as shown in [8]. Then, we use the fact that $\Pi(\zeta,\eta)=\Pi(\eta,\zeta)$. In particular, we find for the phase function of ice fractals: $A=1.247$, $B=1.186$, and $C=5.157$. These coefficients differ from the coefficients for the isotropic scattering. Therefore, the reflection functions for correspondent media are considerably different as well.

The accuracy of Eq. (6) with $\bar{R}$ substituted by the approximate solution given Eq. (9) at $A = 1.247$, $B = 1.186$, $C = 5.157$ as compared to the exact calculations with account for these constants is shown in Fig. 1. The phase function in Eq. (6) was found using the following parameterization of geometrical optics calculations for fractal particles [9,10] with the effective radius 50 μm and the wavelength 0.55 μm valid outside the diffraction region ($\theta \gg 0$):

$$p(\theta) = 11.1\exp(-0.087\theta) + 1.1\exp(-0.014\theta), \tag{11}$$

where the scattering angle θ is in degrees. Note that we have for the nadir observation: $\theta = 180^\circ - \vartheta_1$. The accuracy of this approximation is rather high for angles θ larger than 10° (see Fig. 2). Therefore, it could also be used in other applications (e.g., for singly scattering media). The phase function at smaller angles can be found using the Fraunhofer diffraction theory [9].

Different points in the vertical direction in Fig. 1 represent the azimuthal dependence of the error. In total, we have calculated 91 azimuthal angles [$\varphi = 0^\circ(2^\circ)180^\circ$] for each incidence angle using the exact radiative transfer code described in [2]. Therefore, Fig. 1 illustrates the influence of the azimuthal harmonics on $R(\zeta, \eta, \varphi)$ (for twice and more times scattered light) also.

As one might expect, errors increase with the observation angle. However, they below 3% for the nadir observation (see Fig. 1a) and incidence angles smaller than 78° ($\zeta \geqslant 0.2$). Generally, the error increases with the observation angle. However, it remains below 10% for all azimuths at $\eta \geqslant 0.9$ (i.e., for the observation angles smaller than 25°) and $\zeta \geqslant 0.2$.

We summarize the error analysis in Fig. 3, where regions I–V with different levels of the accuracy of the parameterization derived are shown. We see that all points inside the right-hand-triangle XOY with legs equal to approximately 75° correspond to errors below 5%. The region with largest errors is in the upper-right corner of the map with incidence angles larger than 45° and large values of observation angles. The conditions, where the accuracy of the parameterization is poor, can be avoided in controlled laboratory experiments. They are also not typical in satellite remote sensing problems involving as a rule near-nadir measurements.

We emphasize that reflection functions for media with irregularly shaped particles (e.g., snow) and spheres (e.g., water clouds) differ considerably. This is shown in Fig. 4. In particular, solid lines give the results of exact calculations of the reflection function for particulate semi-infinite nonabsorbing media with spherical and irregularly shaped particles, respectively. It was assumed that the observation is performed in the nadir geometry and spherical particles are characterized by the gamma particle size distribution with the effective radius 6 μm and the coefficient of variance 38% [9]. The wavelength is assumed to be equal to 0.55 μm. It follows that reflection functions for spheres differ from that for fractals considerably. In particular, we see the enhanced values of the reflection at the rainbow and glory geometries for spheres (incidence angles around 40° and 0°, respectively). These features do not exist for fractals. Fractal particles are characterized by larger reflectance for incidence angle larger than 50° at the nadir observation conditions as compared to media composed of spherical particles. This can be easily understood taking into account that a scattering layer with irregularly shaped particles should be generally closer in its reflective properties to the Lambertian reflector assumption ($R \equiv 1$) as compared to media composed of spherical scatterers.

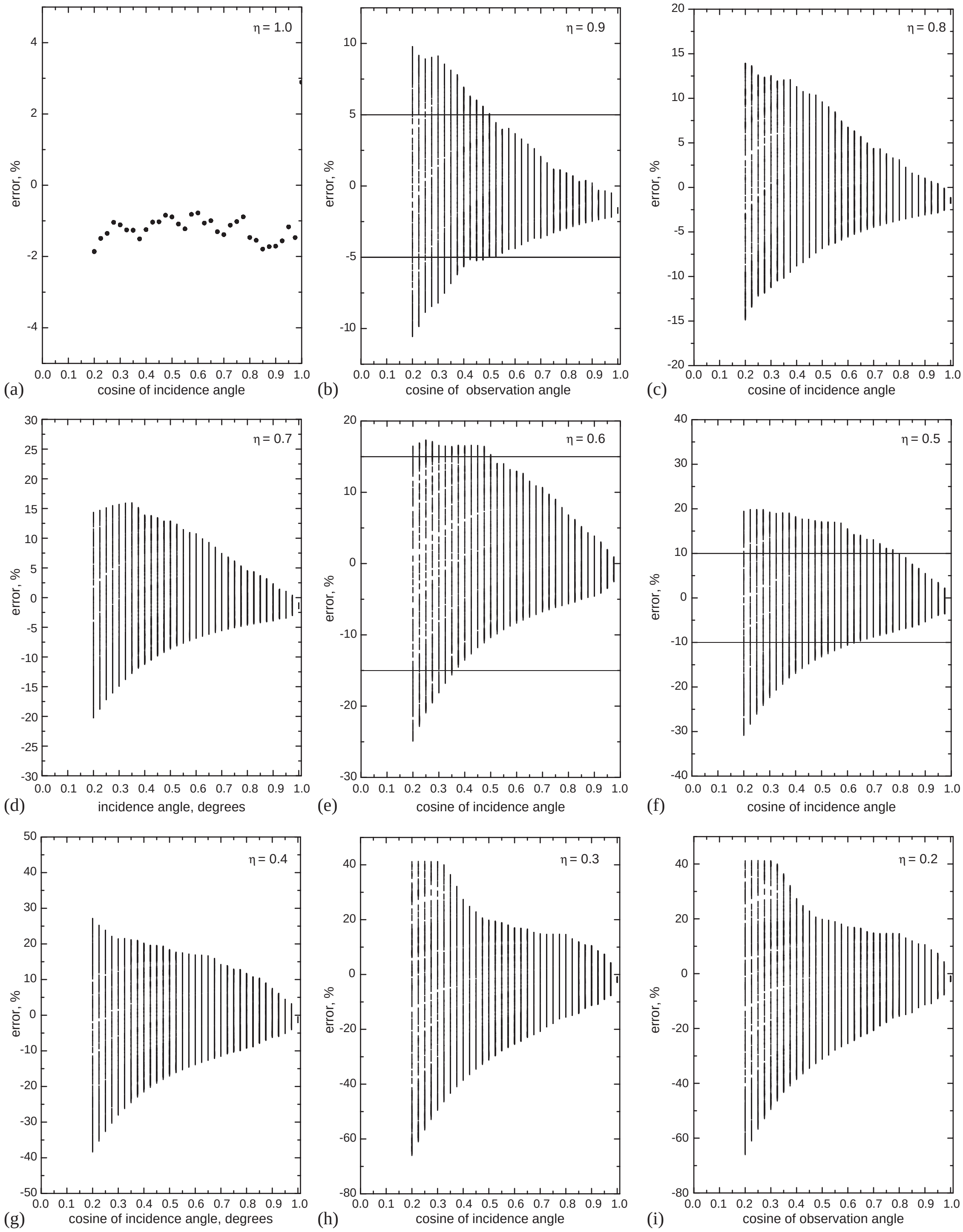

Fig. 1. The error of Eq. (6) at $\eta = 1(a)$, 0.9(b), 0.8(c), 0.7(d), 0.6(e), 0.5(f), 0.4(g), 0.3(h), and 0.2(i). The vertical spread is due to different azimuths used in calculations.

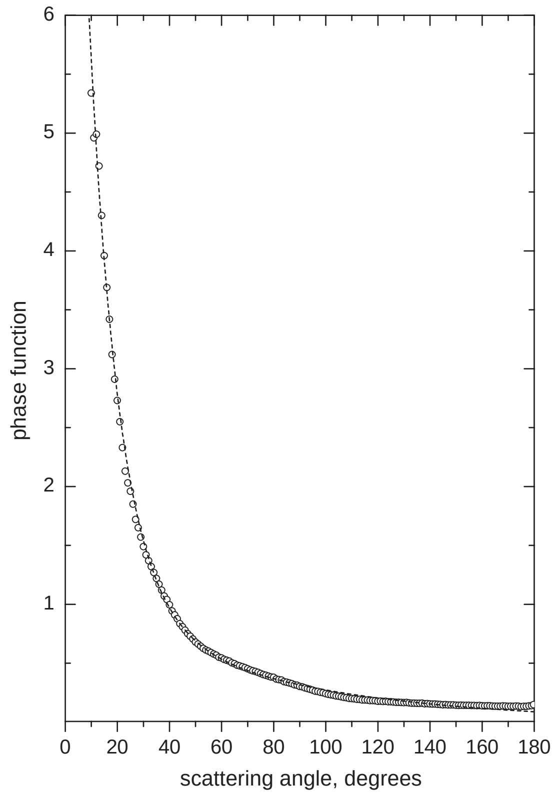

Fig. 2. The phase function of ice fractal particles (see [2] for details) at scattering angles larger than 10°.

Line 1 shows the results of the calculation using Eqs. (6), (9), and (11) at $A = 1.247$, $B = 1.186$, and $C = 5.157$. Line 2 gives the results of calculations using our parameterization but with ignored single scattering contribution (the second term in Eq. (6)). We see that the accuracy of the approximation is high in the nadir case considered even if the single scattering contribution is ignored. This can be easily understood on physical grounds. Light reflection from semi-infinite nonabsorbing media is mostly due to multiple light scattering phenomena.

3. The snow reflection function

It is of importance to compare the derived parameterization not only with the radiative transfer calculations but also with measured reflection functions of snow fields. For this, we present the results of measurements together calculations according to our parameterization in Fig. 5. Measurements have been performed in Hokkaido (Japan) on February 9, 2001 at the wavelength 380 nm for different azimuthal and zenith observation angles (courtesy of T. Aoki). The solar angle was 54°. The details of the measurement technique and instrumentation are given in [11].

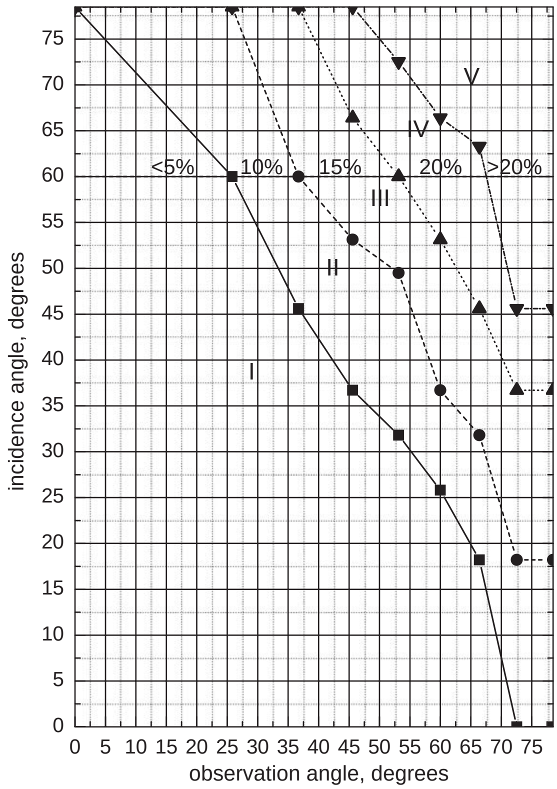

Fig. 3. The error map.

We see that the measured azimuthal dependence is weak (within 5% of the value of the reflection function at $\varphi = 0°$) for azimuths larger than 90° and all observation angles. The minimum at the observation angle close to 60° and $\varphi = 180°$ is caused by the shadow of the instrument.

The thick solid line in Fig. 5 corresponds to the parameterization developed in this paper (see Eq. (6)) with single scattering contribution ignored. The account for the second term in Eq. (6) does not change the results considerably, producing only slight increase in the reflection function for smaller azimuthal angles at larger values of ϑ_2. This tendency is also seen in experimental data.

The analysis of data given in Fig. 5 allows to arrive to the following conclusions. First of all, the parameterization is capable of describing the weak dependence of the reflection function on the azimuth for azimuthal angles larger than 90° and observation angles smaller than 80°. Secondly, the error of the parameterization increases for smaller azimuths (and observation angles larger than 30°). Fortunately, in many cases (e.g., in satellite observations) mostly near—nadir measurements are used. Then the parameterization can be applied for any azimuths outside the sun glint region. Finally, we note that the differences in the measured and calculated reflection functions at the nadir observation (see Fig. 5) may be caused by aerosols (e.g., soot) deposited in snow. These effects are not incorporated in the model proposed.

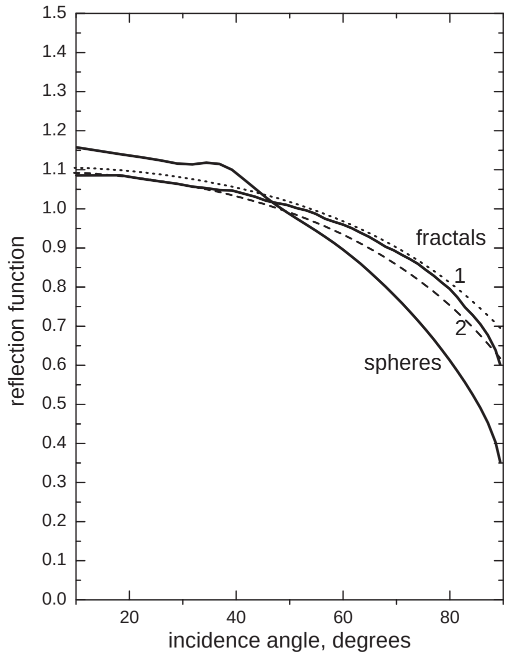

Fig. 4. The reflection function of a semi-infinite nonabsorbing light scattering medium with spherical water droplets as compared to the same media except with fractal scatterers (solid lines). Line 1 shows the reflection function calculated using Eqs. (6), (9), and (11) at $A = 1.247$, $B = 1.186$, and $C = 5.157$. Line 2 corresponds to results obtained ignoring the second term in Eq. (6). The nadir observation is assumed. Further explanations are given in the text.

4. Conclusion

The simple analytical equation for the reflection function of a semi-infinite nonabsorbing medium with irregularly shaped particles is proposed. The parameterization depends on the phase function of the medium and three constants A, B, and C. These constants were found for a particular case of ice fractal particles. However, we believe that equation derived can also be used to account for the angular patterns of the reflected light of turbid media other than snow fields and optically thick ice clouds. This is confirmed by the fact that the similar expression (but with other coefficients) remains valid for media composed of spherical droplets [9].

It was found that the error of the parameterization is below 3% for the nadir observation and incidence angles smaller than 78°. The error increases for oblique incidence/observation conditions (see Fig. 1).

The account for light absorption and a finite thickness of a light scattering layer can be performed in the way specified in [12].

The parameterization of the phase function given by Eq. (11) can be used to study light propagation in snow and ice clouds having an extremely complex structure of scatterers. Then the Mie theory is not valid and various models should be used to represent light scattering by a local volume of a turbid layer. The model of fractal particles has been appeared to be quite reasonable in this extreme case [2].

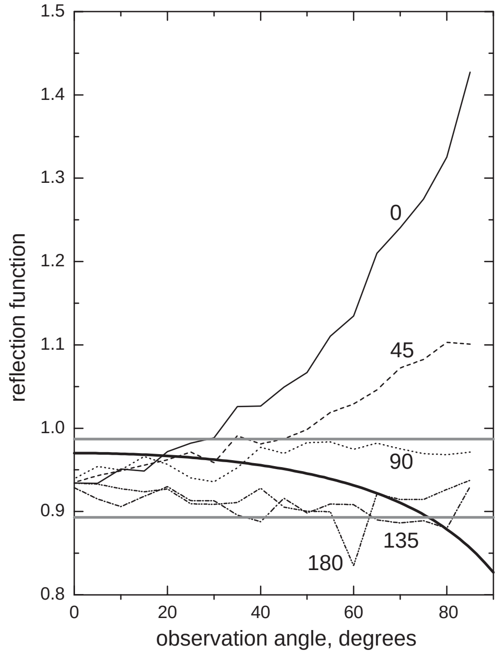

Fig. 5. The measured reflection function of a snow field as the function of the observation angle for azimuths 0°, 45°, 90°, 135°, and 180° at the wavelength 380 nm. The solar zenith angle is equal to 54°. The thick line gives the results obtained using the parameterization (6) with the assumption that the second term is neglected. Grey lines show ±5% deviation from the reflection function at the nadir observation.

The comparison of our parameterization with in situ observations of light reflectance from snow fields shows its great potential as far as remote sensing applications are of concern. It should be pointed out that the model developed has no free parameters for snow fields.

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