1 Is scaling the noise appropriate as a fit parameter?

The solution for the OE problem is given as:

$$\hat{x} = x_a + (S_a^{-1} + K^T S_\epsilon^{-1} K)^{-1} K^T S_\epsilon^{-1}(y - KX_a) \tag{1}$$

hence, the contribution function matrix is given as

$$D = (S_a^{-1} + K^T S_\epsilon^{-1} K)^{-1} K^T S_\epsilon^{-1}. \tag{2}$$

Assume we enhance the SNR by the factor η: $S_\eta = 1/\eta S_\epsilon$.

$$\begin{aligned}
& (S_a^{-1} + K^T \frac{1}{\eta} S_\eta^{-1} K)^{-1} K^T \frac{1}{\eta} S_\eta^{-1} & (3)\\
= \;& (\frac{1}{\eta}(\eta S_a^{-1} + K^T S_\eta^{-1} K))^{-1} K^T \frac{1}{\eta} S_\eta^{-1} & (4)\\
= \;& \eta(\eta S_a^{-1} + K^T S_\eta^{-1} K)^{-1} K^T \frac{1}{\eta} S_\eta^{-1} & (5)\\
= \;& (\eta S_a^{-1} + K^T S_\eta^{-1} K)^{-1} K^T S_\eta^{-1} & (6)\\
= \;& (S_b^{-1} + K^T S_\eta^{-1} K)^{-1} K^T S_\eta^{-1}. & (7)
\end{aligned}$$

with $S_b = \frac{1}{\eta} S_a$. Therefor, scaling the signal to noise ratio by a factor $\sqrt{\eta}$ is the same as scaling the apriori covariance by a factor of $\frac{1}{\sqrt{\eta}}$, or improving the SNR is equal to narrowing down the apriori covariance.

This can be understood by looking at the covariance S of the a posteriori: The original covariance is:

$$\hat{S} = (S_a^{-1} + K^T S_\epsilon^{-1} K)^{-1} \tag{8}$$

The covariance gets smaller when increasing the SNR (i.e. the inverse of the noise).

Putting it differently: The result is assumed to be fixed. If the SNR gets better, that is, there is more information in the signal, the force to drive $\hat{x}$ towards the true value becomes greater, hence, the apriori covariance should be chosen stronger in order to get the same result as before.

2 Effects of wrong SNR

Using the result above, we can judge the effects of a given SNR which is either to good or to bad.

Let S_ϵ denote the true signal to noise ratio of a given spectrum. The assumed SNR is obtained by scaling by η. If assumed the SNR is to good, i.e., η is increased the measurement becomes more weight and the result is likely to be wrong unless the a priori covariance was very narrow before (weight put on a priori).

3 Conclusions

- Scaling the measurement noise and the a priori covariance is equivalent.
- Fine tuning of the retrieval for different information content in different altitudes, the SNR is not an adequat fitting parameter.
- If the retrieval is optimized, overestimating the SNR is likely to lead to wrong results (the noise is being fitted).
- Underestimating the SNR would lead to wasting of information.