Digital Image Processing, 2017

Exercise 7

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Problem 22: Products of vectors

By $\boldsymbol{x}$ we understand a column vector, $\boldsymbol{x}^{\mathrm{T}}$ is a row vector. If $\boldsymbol{x}^{\mathrm{T}} = (1, 2, 3)$, what is $\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}$, what is $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$?

Problem 23: Covariance Matrix

Determine the covariance matrix C_x for the following set of points in three dimensions: $\boldsymbol{x_a} = (0, 0, 0)^{\mathrm{T}}$, $\boldsymbol{x_b} = (1, 0, 0)^{\mathrm{T}}$, $\boldsymbol{x_c} = (1, 1, 0)^{\mathrm{T}}$, $\boldsymbol{x_d} = (1, 0, 1)^{\mathrm{T}}$. $\boldsymbol{x_e} = (2, 2, 0)^{\mathrm{T}}$. $\boldsymbol{x_f} = (3, 2.5, 0)^{\mathrm{T}}$. Note that the covariance matrix is a 3×3 matrix.

What do the elements of C_x tell us?

Problem 24: Principal Component Analysis

Show that …

a) …for the mean values of the vectors $\boldsymbol{y}$ after the principal components transformation we have: $\boldsymbol{m}_y = 0$.

b) …the inverse of the transformation matrix is $W^{-1} = W^{\mathrm{T}}$. Hint: Do this by showing that WW^{T} is the identity matrix. Remember that the rows of W (and thus the columns of W^{T}) consist of orthonormal vectors.