Digital Image Processing, 2017

Exercise 8

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June 8, 2017
submit by next Wedneaday, 12:00

Problem 25: Shifting Property

Prove the shifting property of Fourier transforms, i.e.,

$$\text{if} \quad \mathcal{FT}[f(x)] = F(u) \quad \text{then} \quad \mathcal{FT}[f(x-a)] = \exp(-\mathrm{i}2\pi ua)F(u)$$

Problem 26: Scaling Property

a) Prove the scaling property of Fourier transforms, i.e.,

$$\text{if} \quad \mathcal{FT}[f(x)] = F(u) \quad \text{then} \quad \mathcal{FT}[f(\frac{x}{b})] = |b|F(bu)$$

b) Use this to calculate the Fourier transform of a rectangle function of width b. How does the Fourier transform look for large b and for small b (a rough sketch might help).

Problem 27: Fourier Transform of Complex Exponential and Sinusoidal

Determine, without evaluating any integrals, the Fourier transform of

a) the constant function $f(x) = b$. Hint: Use the fact that $\mathcal{FT}[\exp(2\pi\mathrm{i}ax)] = \delta(u-a)$.

b) $\cos(2\pi ax)$. Hint: $\cos(x) = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}\right)$. What does the result have to do with the following illustration (Figure 1)?

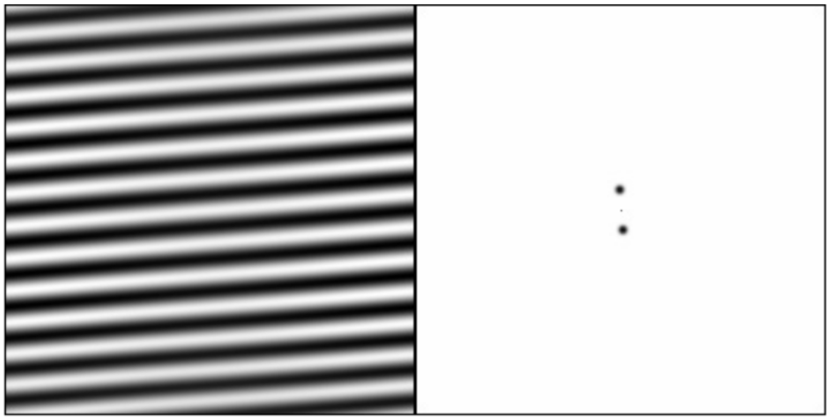

Figure 1: Left half: an image; right half: its Fourier transform

Problem 28: Fourier Transform of Triangle Function

Calculate the Fourier transform of the triangle function

$$\mathrm{tri}(x) = \begin{cases} 1 - |x| & : \quad |x| \le 1 \\ 0 & : \quad \text{else} \end{cases}$$

without evaluating any integrals. Use the fact that $\mathrm{tri} = \mathrm{rect} * \mathrm{rect}$ (convolution of rectangle with itself).

Problem 29: Filtering in Fourier Domain

Figure 2 shows an image (it is a wasp's head) and its Fourier spectrum (absolute square of its Fourier transform). The five superimposed circles on the spectrum enclose 90, 93, 95, 99 and 99.5% of the image power, respectively (the innermost circle is hardly visible. Figure 3(b)–(f) shows five filtered versions of the same image. What type of filter was used, and what is the difference between the different filtered images? Note: The circles have something to do with it.

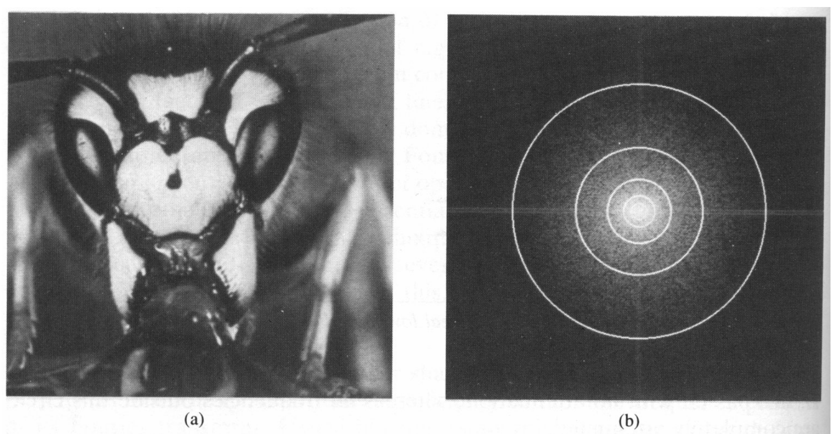

Figure 2: Left: An image. Right: Its Fourier spectrum (absolute square of its Fourier transform; black represents zero), with five superimposed circles.

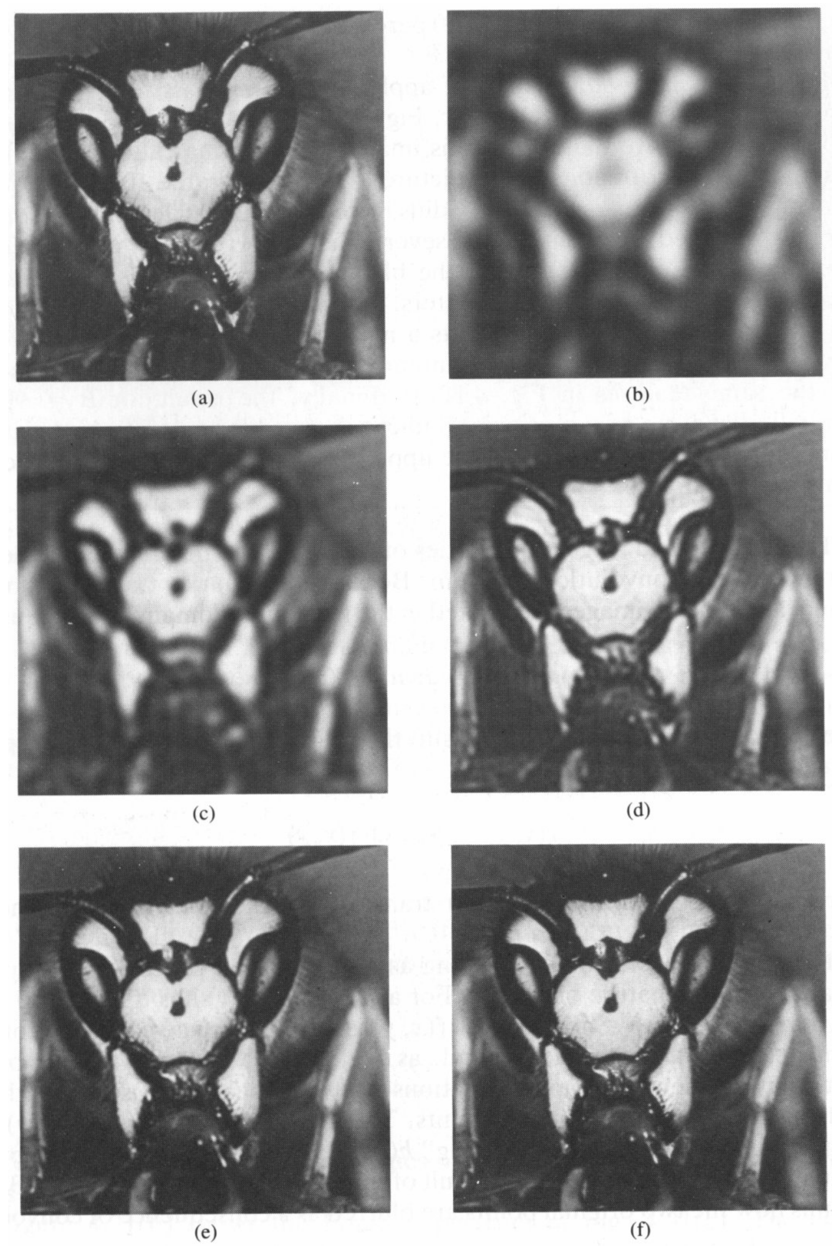

Figure 3: (a) Original image, same as in previous figure, (b)–(f) filtered versions of that image.